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National Advertising and Cooperation in a Manufacturer-Two Retailers Channel

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ABSTRACT: We consider a supply channel composed of one manufacturer and two symmetric retailers. Three cases are studied. The non-cooperation case is a leader-follower relationship. The manufacturer determines his spending in national advertising and the wholesale price. Then, retailers determine non-cooperatively the price for consumers. In the partial-cooperation case, retailers decide jointly for the price. In the full-cooperation case, all members of the channel cooperate by maximizing a joint profit function. Interestingly, partial-cooperation reduces the profits of retailers with respect to non-cooperation, when the degree of substituability between the two products proposed by retailers is low. Because of symmetry, this also implies that the total profit of retailers may decrease with partial-cooperation. Thus, when the degree of substituability between products is low, it is in the interest of retailers to set their prices non-cooperatively. We propose a cooperative implementable contract between all channel members, which shares the extra-profit due to full-cooperation. We propose a new and unusual evaluation of consumers' surplus which positively depends not only on the price-demand function but also on the spending in national advertising. Partial-cooperation is always the worst case for the manufacturer, the whole channel, consumers' surplus and social welfare, while full-cooperation is the best case.

Keywords: Game theory; National advertising; Partial-cooperation; Full-cooperation; Welfare
JEL Classifications: C70; D60; M30

1. Introduction

Supply channel management research has gained a considerable attention. One interesting issue in this area is how the actions taken by one member of the channel can influence the profitability of other members. The supply channel implies an important relationship between its different members as manufacturers and retailers. This relationship can be non-cooperative or cooperative. In the non-cooperative situation, each member of the supply channel has his own objective function. The members who act first, usually manufacturers, are leaders, while those who react, as retailers, are followers. In a cooperative situation, the channel members work together for the same goal.

Many studies on advertising efforts and pricing policy have focused on distribution channels formed by one manufacturer and one retailer (Karray and Zaccour, 2006; Yue *et al.,* 2006; Xie and Neyret, 2009; SeyedEsfahani *et al.,* 2011). Xie and Wei (2009) addressed channel coordination by seeking optimal cooperative advertising strategies and equilibrium pricing in a manufacturer-retailer distribution channel. They compared two models: one is a non-cooperative leader-follower game, where the leader is the manufacturer and the follower is the retailer, and the other is a cooperative game. They showed that cooperative model achieves better coordination by generating higher channel total profit, lower retail price to consumers, and higher advertising efforts for all channel members than the non-cooperative model. They identified the feasible solutions to a bargaining problem where the channel members can determine how to divide the extra-profits generated by cooperation.

Other papers have been interested in distribution channels without advertising (Choi, 1996; Tsay and Agrawal, 2004; Ingene and Parry, 2004, 2007; Cachon and Lariviere, 2005; Xiao and Qi, 2008). Yang and Zhou (2006) considered the pricing and quantity decisions of a two echelon system with a manufacturer who supplies a single product to two competitive retailers. A Stackelberg structure is assumed where the manufacturer is a leader and the retailers are followers. They analyzed the effects of the duopolistic retailers' different competitive behaviors (Cournot, Collusion and Stackelberg) on the optimal decisions of the manufacturer and the retailers.

Taboubi and Zaccour (2005) reviewed the major contributions in the literature that examined the issue of channel coordination according to the game theory approach. They highlighted the main literature results and identified research questions for further investigation. Karray and Zaccour (2007) considered a distribution channel formed by two manufacturers and two retailers to investigate whether cooperative advertising programs are profitable to such channels. They showed that, under some conditions, cooperative advertising may be profitable to retailers and the whole channel, but not to the manufacturers. However, their model is limited to local advertising with no national advertising, and full cooperation between the retailers or between all channel members are not studied. Wang *et al.,* (2009) found that, when the company store and the independent retailer compete in the same market, the former charges higher price and provides more marketing effort. Wang *et al.,* (2011) considered a distribution channel formed by one manufacturer and two retailers. They discussed four possible game structures: Stackelberg-Cournot, Stackelberg-collusion, Nash-Cournot and Nash-collusion. They revealed how cooperative advertising policies and profits of all participants are affected by various competitive behaviors. However, all their results are made under the heavy assumption that the retailers and the manufacturer's marginal profits are exogenously determined. Moreover, welfare implications are not studied.

Our research is closely related to that of Xie and Wei (2009). We made some simplifications to their model by considering that there are no retailers' local advertising expenditures and no manufacturer's participation rate. However, we enrich their model by considering two competing retailers. This extension enables us to study the case of cooperation between retailers. In addition, we evaluate the impact of cooperation between retailers and between all members of the supply channel on consumers' and social welfare. Such welfare consequences are interesting and have not been done before by previous studies on supply channel.

We consider a supply channel game model with a single manufacturer and two symmetric retailers. The manufacturer sells a product with a wholesale price to retailers, which sell the product purchased to final consumers. Without loss of generality, we suppose that production and handling costs are zero. The manufacturer uses national advertising to increase consumers' interest in the product. For tractability reasons, we suppose that there are no local advertising expenditures for retailers, and no participation rates from the manufacturer to retailers. Consumers' effective-demand for the product depends not only on its price-demand, i.e. the demand due to the product price, but also on the advertisement effort made. The manufacturer determines its wholesale price and national advertising spending. Then, retailers determine the price for consumers. We consider and compare three cases. The non-cooperative case, where the manufacturer and retailers decide non-cooperatively, and each of them maximizes its own profit. The partial-cooperation case, where only the two retailers cooperate by maximizing a joint profit function. Finally, in the full-cooperation case, the three members of the supply channel engage in a cooperative program and maximize the total profit of the supply channel. We note that for the first two cases, the manufacturer is the leader while the retailers are followers. These games are solved backward to obtain subgame-perfect Nash equilibriums.

We show that when only retailers cooperate, this has no effect on the wholesale price, the price for consumers is the highest, while the quantity produced and the spending in advertising are the lowest with respect to non-cooperation and full-cooperation.

Interestingly, when the degree of substituability between the two products proposed by retailers is not important, partial-cooperation deteriorates the profit of each retailer and, because of symmetry, deteriorates the total profit of retailers. Thus, when the degree of substituability between products is low, it is in the interest of retailers to set their prices non-cooperatively. This is the principal result of our paper. This result is new and non-obvious for symmetric retailers, because retailers are usually better off when they cooperate. This result is due to the spending in national advertising. Indeed, when retailers unilaterally cooperate, the retail price increases and the spending in national advertising decreases leading to an important decrease in sales and in retailers' profits.

Let's notice that when retailers are symmetric, Yang and Zhou (2006) showed that each partially-cooperating retailer gains more than with non-cooperation. However, when retailers are asymmetric and face different demand functions, one retailer may be worse off, whereas the other may be better off, with partial-cooperation. The total profit of retailers being always higher with partial-cooperation. These results are intuitive. Wang *et al.* (2011) showed that partially-cooperating retailers can be worse off under some conditions for both symmetric special case and Stackelberg game. However, their findings are obtained under heavy hypotheses: the marginal profits per unit of retailers and manufacturer, and consequently the selling prices of retailers, are exogenously determined.

When the degree of substituability between the two products is sufficiently high, we have the standard result that partial-cooperation of retailers increases their profits with respect to non-cooperation. This retailers' collusion on price is sustainable because, in our paper, it leads to a symmetric equilibrium and equal gains for the two retailers. Moreover, partial-cooperation decreases the profit of the manufacturer because the wholesale price is not affected and the quantities sold are diminished. Consequently, and independently of whether the degree of substituability between the two goods is high or low, the total profit of the supply channel is the lowest when only retailers cooperate.

When all members of the supply channel cooperate, the price for consumers is the lowest, the spending in advertising, production and total profit of the supply channel are the highest. The manufacturer and retailers can determine a wholesale price enabling them to share this extra-profit due to full-cooperation. When this cooperative wholesale price is at its lower bound, all the extra-profit goes to retailers; when it is at its higher bound, all the extra-profit goes to the manufacturer; and when it is in the middle, the extra-profit is equally shared by the manufacturer and the two retailers. We design a contract making this full-cooperative outcome implementable with a reasonable wholesale price.

We propose a new and unusual evaluation of consumers' surplus which does not depend only on the price-demand function, but it also depends positively on the spending in national advertising. We show that consumers' and social welfare are the lowest when only retailers cooperate, and they are the highest when all members of the channel cooperate. This constitutes an interesting and non-obvious contribution of our paper since previous studies have not evaluated the impact of cooperation on consumers' and social welfare.

The paper is organized as follows. Section 2 presents our basic game-theoretic model. Section 3 solves the non-cooperation case. Section 4 solves the partial-cooperation game. Section 5 solves the full-cooperation case. Section 6 studies the extra-profit sharing. Section 7 discusses and compares the three cases studied, and Section 8 concludes.

2. The Basic Model

We consider a manufacturer-two-retailers distribution channel in which both retailers sell only the manufacturer's brand within the product class. Decision variables for the manufacturer are the national advertising expenditure A and the wholesale price to retailers[1] w. The decision variables for the retailers are their retail prices p_i , $i = 1, 2$. For tractability reasons, we suppose that there are no local advertising expenditures for retailers, and no participation rates from the manufacturer to retailers. This is a leader-follower game: the manufacturer chooses his decision variables, and then retailers choose their retail prices. This game is solved backward to get a subgame-perfect Nash equilibrium. Let's notice that our model is not suitable for the Nash game or the Stackelberg-retailers game.

The manufacturer uses brand advertising to increase consumers' interest and demand for the good produced. Consumers' demand V_i, or effective-demand, for the good proposed by retailer i, also known as the sale response function, depends on retail prices and the advertising level in a multiplicative manner as many studies (Xie and Wei, 2009; Yue *et al.*, 2006):

$$V_i(p_i, p_j, A) = g_i(p_i, p_j)\, h(A), i = 1,2, j = 3 - i \quad (1)$$

[1] The Robinson-Patman Act requires comparable treatment of competing retailers (Moorthy, 1987).

where $g_i(p_i, p_j)$ and $h(A)$ reflect the impact of retail prices and the brand advertising expenditures on the demand of product i, respectively. To distinguish between the effective demand and the demand due to price variations, we will call $g_i(p_i, p_j)$ as the price-demand function for product i.

As many studies (Yang and Zhou, 2006; Ingene and Parry, 2007; Xiao and Qi, 2008), we assume that the price-demand function for product i is linear with retail prices:[2]

$$g_i(p_i, p_j) = 1 - p_i + \beta p_j, 0 < \beta < 1, i = 1,2, j = 3 - i \tag{2}$$

where β is the degree of substituability between the two products proposed by retailers. The maximum value for $g_i(p_i, p_j)$ is normalized to 1 for simplicity of expressions.

The impact of national advertising expenditures on the effective-demand of product i is an increasing and concave function consistent with the advertising saturation effect:[3]

$$h(A) = \sqrt{A} \tag{3}$$

Therefore, we have:

$$V_i(p_i, p_j, A) = (1 - p_i + \beta p_j)\sqrt{A}, i = 1,2, j = 3 - i \tag{4}$$

We suppose that both the manufacturer's unit production cost and retailers' unit handling cost are constant. We normalize them to zero to simplify our expressions.

The profits of the manufacturer, each retailer, the two retailers, and the whole system are, respectively:

$$\pi_m = w\,(V_1 + V_2) - A \tag{5}$$

$$\pi_{r_i} = (p_i - w)V_i \tag{6}$$

$$\pi_{r_1+r_2} = (p_1 - w)V_1 + (p_2 - w)V_2 \tag{7}$$

$$\pi_t = \pi_m + \pi_{r_1} + \pi_{r_2} = p_1V_1 + p_2V_2 - A \tag{8}$$

An important contribution of this paper is the evaluation of the impact of cooperation between retailers and between all members of the channel on consumers' and social welfare.

Consumers' surplus engendered by the consumption of quantity V_i of the product sold by retailer i is:

$$CS(V_i) = \int_0^{V_i} p_i(t)\,dt - p_iV_i \tag{9}$$

From (4), we have:

$$p_i(V_i) = 1 + \beta p_j - \frac{V_i}{\sqrt{A}}, i = 1, 2, j = 3 - i \tag{10}$$

Using (10) in (9), we get:

$$CS(V_i) = \frac{V_i^2}{2\sqrt{A}} = \frac{g_i^2}{2}\sqrt{A} \tag{11}$$

The above expression is a new evaluation of consumers' surplus, or consumers' welfare, which is function of the price-demand for the good and of the spending in advertising. It is a new expression because we are accustomed with consumers' surplus in micro-economic theory in function only of the

[2] Using a more general, symmetric and linear, price-demand function as $g_i(p_i, p_j) = a - \alpha p_i + \beta p_j, 0 < a, 0 < \beta < \alpha$, does not change our analytical results.

[3] We can use a more general function $h(A) = l\sqrt{A}, l > 0$, but this has no effect on our analytical results.

price-demand. This expression shows that consumer' surplus increases with the price-demand for the good, which is an usual result, and also increases with the national advertising spending, which is a new result.

Total consumers' surplus, i.e. consumers' welfare, engendered by the consumption of the two products is:

$$CS_t = CS(V_1) + CS(V_2) \tag{12}$$

We define the social welfare as total consumers' surplus plus the total profit of the supply channel:

$$S = CS_t + \pi_t \tag{13}$$

In what follows, we will solve backward the three games.

3. The Non-Cooperation Game

The three members of the supply channel behave non-cooperatively. It is a two-stage game. In the first stage, the manufacturer (leader) maximizes his profit with respect to its decision variables, which are w and A. Then, each retailer (follower) maximizes his profit function with respect to the price he proposes for consumers.

Solving the second-stage first-order conditions, which are $\frac{\partial \pi_{r_i}}{\partial p_i} = 0$, i =1, 2, gives the retail prices, which are symmetric:

$$p_i^* = p^* = \frac{1+w}{\delta} \tag{14}$$

where $\delta = 2 - \beta$. We can verify that $1 < \delta < 2$.

Using the expression given by (14) in (5), we get:

$$\pi_m^* = \frac{2}{\delta} w(1 - \lambda w)\sqrt{A} - A \tag{15}$$

where $\lambda = 1 - \beta$, verifies $0 < \frac{\lambda}{\delta} < \frac{1}{2}$.

Using (15) and solving the first-stage first-order conditions[4] for the manufacturer, which are $\frac{\partial \pi_m^*}{\partial w} = 0$ and $\frac{\partial \pi_m^*}{\partial A} = 0$, we get the optimal wholesale price and advertising spending:

$$w^* = \frac{1}{2\lambda} \tag{16}$$

$$A^* = \frac{1}{16\lambda^2\delta^2} \tag{17}$$

Using $w^* = \frac{1}{2\lambda}$ and $A^* = \frac{1}{16\lambda^2\delta^2}$ in the other expressions, we get the optimal values for the non-cooperation case of the other variables, which are given in Table 1. It is easy to verify that the wholesale price is lower than the retailers' price. Also, we can verify that $\pi_m^* > 2\pi_r^*$, meaning that the manufacturer gains more than the two retailers together.

4. The Partial-Cooperation Game

In this section, retailers decide to cooperate by maximizing their joint profit function, while the manufacturer still maximizes his own profit function. This is a two-stage game where the manufacturer plays first (leader) and retailers play second (followers).

Solving the second-stage first-order conditions,[5] which are $\frac{\partial \pi_{r_1+r_2}}{\partial p_i} = 0, i = 1, 2,$ gives retail prices, which are symmetric:

[4] Second-order conditions are verified because $\frac{\partial^2 \pi_m^*}{\partial w^2} < 0, \frac{\partial^2 \pi_m^*}{\partial A^2} < 0$ and $\frac{\partial^2 \pi_m^*}{\partial w \partial A} = 0$.

$$\bar{p}_i = \bar{p} = \frac{1 + \lambda w}{2\lambda} \tag{18}$$

Using the expression given by (18) in (5), we get:

$$\bar{\pi}_m = w(1 - \lambda w)\sqrt{A} - A \tag{19}$$

Using (19) and solving the first-stage first-order conditions[6] for the manufacturer, which are $\frac{\partial \bar{\pi}_m}{\partial w} = 0$ and $\frac{\partial \bar{\pi}_m}{\partial A} = 0$, we get the optimal wholesale price and advertising spending:

$$\bar{w} = \frac{1}{2\lambda} \tag{20}$$

$$\bar{A} = \frac{1}{64\lambda^2} \tag{21}$$

Using the optimal values of the decision variables, we get the optimal values for the partial-cooperation case of the other variables, which are given in Table 1.

We can verify that the wholesale price is lower than the retail price. It is easy to verify that $\bar{\pi}_m = 2\bar{\pi}_r$. Contrary to the non-cooperation case, when retailers cooperate, their joint gain is equal to that of the manufacturer.

5. The Full-Cooperation Game

In this case, the manufacturer and retailers agree to make decisions that maximize the total supply channel profit. Then, they negotiate how they will share the extra-profit engendered by such cooperation.

The total profit of the system given by (8) can be written as:

$$\pi_t = (p_1 + p_2 - p_1^2 - p_2^2 + 2\beta p_1 p_2)\sqrt{A}\text{-}A \tag{22}$$

The total profit of the system depends only on p_1, p_2 and A. The three first-order conditions[7] of optimality are $\frac{\partial \pi_t}{\partial p_1} = 0$, $\frac{\partial \pi_t}{\partial p_2} = 0$ and $\frac{\partial \pi_t}{\partial A} = 0$, which give us the unique cooperative solution, which is symmetric:

$$p_i^c = p^c = \frac{1}{2\lambda} \tag{23}$$

$$A^c = \frac{1}{16\lambda^2} \tag{24}$$

In Table 1, we give the cooperative values of the remaining expressions by using the optimal values of the decision variables.

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[5] Second-order conditions are verified because $\begin{vmatrix} \frac{\partial^2 \pi_{r_1+r_2}}{\partial p_1^2} & \frac{\partial^2 \pi_{r_1+r_2}}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi_{r_1+r_2}}{\partial p_1 \partial p_2} & \frac{\partial^2 \pi_{r_1+r_2}}{\partial p_2^2} \end{vmatrix} = \begin{vmatrix} -2\sqrt{A} & 2\beta\sqrt{A} \\ 2\beta\sqrt{A} & -2\sqrt{A} \end{vmatrix} > 0.$

[6] Second-order conditions are verified because $\frac{\partial^2 \bar{\pi}_m}{\partial w^2} = -2\lambda\sqrt{\bar{A}} < 0, \frac{\partial^2 \bar{\pi}_m}{\partial A^2} = -\frac{\bar{w}}{8\bar{A}^{3/2}} < 0$ and $\frac{\partial^2 \bar{\pi}_m}{\partial w \partial A} = 0.$

[7] Second-order conditions are verified by using the following partial derivatives: $\frac{\partial^2 \pi_t}{\partial p_1^2} = \frac{\partial^2 \pi_t}{\partial p_2^2} = -2\sqrt{A}$, $\frac{\partial^2 \pi_t}{\partial p_1 \partial p_2} = 2\beta\sqrt{A}$, $\frac{\partial^2 \pi_t}{\partial A \partial p_1} = \frac{\partial^2 \pi_t}{\partial A \partial p_2} = 0$, $\frac{\partial^2 \pi_t}{\partial A^2} = \frac{-1}{8\lambda A^{3/2}}$.

Table 1. Comparison of results for the three cases[8]

Non-cooperation	**Partial-cooperation**	**Full-cooperation**	**Comparisons**
$w^* = \frac{1}{2\lambda}$	$\bar{w} = \frac{1}{2\lambda}$	$\frac{1+\delta^2}{4\lambda\delta^2} < w^c < \frac{\delta^3 - \lambda}{2\lambda\delta^3}$	$w^c < w^* = \bar{w}$
$p^* = \frac{\lambda+\delta}{2\lambda\delta}$	$\bar{p} = \frac{3}{4\lambda}$	$p^c = \frac{1}{2\lambda}$	$p^c < p^* < \bar{p}$
$A^* = \frac{1}{16\lambda^2\delta^2}$	$\bar{A} = \frac{1}{64\lambda^2}$	$A^c = \frac{1}{16\lambda^2}$	$\bar{A} < A^* < A^c$
$V^* = \frac{1}{8\lambda\delta^2}$	$\bar{V} = \frac{1}{32\lambda}$	$V^c = \frac{1}{8\lambda}$	$\bar{V} < V^* < V^c$
$\pi_r^* = \frac{1}{16\lambda\delta^3}$	$\bar{\pi}_r = \frac{1}{128\lambda^2}$	$\pi_r^c = \frac{1-2\lambda w^c}{16\lambda^2}$	$\bar{\pi}_r < \pi_r^* \Leftrightarrow \beta < 3 - \sqrt{5}$
$\pi_m^* = \frac{1}{16\lambda^2\delta^2}$	$\bar{\pi}_m = \frac{1}{64\lambda^2}$	$\pi_m^c = \frac{4\lambda w^c - 1}{16\lambda^2}$	$\bar{\pi}_m < \pi_m^*$
$\pi_t^* = \frac{2\lambda+\delta}{16\lambda^2\delta^3}$	$\bar{\pi}_t = \frac{1}{32\lambda^2}$	$\pi_t^c = \frac{1}{16\lambda^2}$	$\bar{\pi}_t < \pi_t^* < \pi_t^c$
$CS_t^* = \frac{1}{16\lambda\delta^3}$	$\overline{CS}_t = \frac{1}{128\lambda}$	$CS_t^c = \frac{1}{16\lambda}$	$\overline{CS}_t < CS_t^* < CS_t^c$
$S^* = \frac{3\lambda+\delta}{16\lambda^2\delta^3}$	$\bar{S} = \frac{4+\lambda}{128\lambda^2}$	$S^c = \frac{\delta}{16\lambda^2}$	$\bar{S} < S^* < S^c$

6. Extra-Profit Sharing

To commit to a cooperative program, the profits of the manufacturer and retailers through full-cooperation should be higher than their own profits realized in the non-cooperation Stackelberg game. We need a bargaining mechanism to motivate the channel members to cooperate and to share the extra-profit engendered by full-cooperation, which is:

$$\Delta\pi_t = \pi_t^c - \pi_t^* > 0 \tag{25}$$

To share this extra-profit due to cooperation, the members of the channel can set a wholesale price w^c for each unit of product purchased by retailers from the manufacturer. Let's notice that in Wang *et al.* (2011) the sharing of the extra-profit is done by means of the fraction of local advertising costs paid by the manufacturer.

Using expression (5) with production and advertising spending equal to $V^c$and A^c, respectively, the profit of the manufacturer under full-cooperation is:

$$\pi_m^c = \frac{4\lambda w^c - 1}{16\lambda^2} \tag{26}$$

The manufacturer will participate to full-cooperation iff

$$\pi_m^c > \pi_m^* \Leftrightarrow w^c > w_{min}^c = \frac{1+\delta^2}{4\lambda\delta^2} \tag{27}$$

[8] Almost all comparisons are easy to establish. We present some of them: *i)* $\bar{\pi}_r < \pi_r^* \Leftrightarrow \beta^2 - 6\beta + 4 = [\beta - (3-\sqrt{5})][\beta - (3+\sqrt{5})] > 0 \Leftrightarrow \beta < 3 - \sqrt{5}$.

ii) $\bar{\pi}_t < \pi_t^* \Leftrightarrow 6(1-\beta) + \beta^2 > 0$: this is true.

iii) $\pi_t^* < \pi_t^c \Leftrightarrow 6\beta^2 - \beta^3 - 9\beta + 4 = 4(1-\beta)^2\left(1 - \frac{\beta}{4}\right) > 0$: this is evident.

Thus, if the wholesale price is higher than w_{min}^{c}, the manufacturer finds full-cooperation interesting.

Using expression (6) with retail prices and expenditures in advertising equal to p^c and V^c, respectively, the profit of each retailer under full-cooperation is:

$$\pi_r^c = \frac{1 - 2\lambda w^c}{16\lambda^2} \tag{28}$$

Non-cooperating retailers will participate in the full-cooperation game iff

$$\pi_r^c > \pi_r^* \iff w^c < w_{max}^c = \frac{\delta^3 - \lambda}{2\lambda\delta^3} \tag{29}$$

Therefore, when the wholesale price does not exceed a certain value w_{max}^c, it is in the interest of non-cooperating retailers to cooperate with all members of the supply channel. Thus, we can establish the following Proposition:

Proposition 1. *To get all partners interested in cooperation, the wholesale price should be between a minimal value and a maximal value:*

$$w_{min}^c < w^c < w_{max}^c \tag{30}$$

We can easily verify that $w_{min}^c < w_{max}^c$ and that, when inequality (30) is verified, then $w^c < p^c$.

The above Proposition shows that, when the retail prices and the national advertising spending are set at their cooperative values $\mathrm{p^c}$ and $\mathrm{A^c}$and the wholesale price belongs to $]\mathrm{w_{min}^c}, \mathrm{w_{max}^c}[$, cooperating channel members are better than with non-cooperation. A cooperative implementable contract between all channel members means that each retailer buys from the manufacturer the quantity $\mathrm{V^c}$ at the wholesale price $\mathrm{w^c}$, and sells it to consumers at price $\mathrm{p^c}$. The manufacturer engages himself to spend $\mathrm{A^c}$ in national advertising.

A wholesale price near $\mathrm{w_{min}^c}$ gives a higher share of the extra-profit to retailers, and when it is near $\mathrm{w_{max}^c}$, it gives a higher share to the manufacturer. When $\mathrm{w^c} = \mathrm{w_{min}^c}$, all the extra-profit goes to retailers: the manufacturer is indifferent between cooperating or not. When $\mathrm{w^c} = \mathrm{w_{max}^c}$, all the extra-profit goes to the manufacturer: retailers are indifferent between full-cooperation and non-cooperation.

Proposition 2. *The wholesale price that splits equally the extra-profit between the cooperating manufacturer and the two retailers is*

$$w_e^c = \frac{\mathrm{w_{min}^c} + \mathrm{w_{max}^c}}{2} \tag{31}$$

Indeed, with $\mathrm{w^c} = w_e^c$, we have $\frac{\Delta\pi_t}{2} = \pi_m^c - \pi_m^*$.

7. Comparison of Results and Discussions

From the comparisons presented in Table 1, we deduce the following Propositions.

Proposition 3. $(i)\, w^c < w^* = \bar{w}, (ii)\, p^c < p^* < \bar{p}, (iii)\, \bar{A} < A^* < A^c, (iv)\, \bar{V} < V^* < V^c.$

With partial-cooperation, the retail price is the highest, whereas the quantity purchased and the spending in advertising are the lowest. Indeed, when retailers cooperate, the retail price increases reducing the price-demand for the product. Because of the multiplicability of the sale response function (see (4)), the spending in national advertising becomes less efficient for the manufacturer, inciting him to reduce advertising spending. The fact that expenditures in advertising are the lowest under partial-cooperation is a new and interesting result. Wang *et al.* (2011, Theorem 4) showed that the spending in national advertising may be higher or lower with partial-cooperation than with non-cooperation, with the assumption that retail prices are exogenously determined.

The above Proposition shows that the wholesale price does not depend on whether retailers cooperate or not. Moreover, the wholesale price of full-cooperation, which is determined to share the extra-profit, is the lowest because the retail price is the lowest with respect to those of non-cooperation and partial-cooperation. Indeed, when all members of the channel cooperate, there is no double marginalization, and the price for consumers is the lowest. Due to the multiplicability of the sale response function, the spending in national advertising becomes more efficient for the manufacturer, inciting him to increase advertising spending. Consequently, the quantity purchased and the spending in advertising are the highest under full-cooperation. This last result concerning national advertising is similar to that of Xie and Wei (2009) who have considered a supply channel composed of only one retailer, whereas Wang *et al.* (2011) have not addressed this question.

From the comparisons in Table 1, we can establish the principal result of this paper:

Proposition 4. *When the degree of substituability between the two products is sufficiently low, (symmetric) partially-cooperating retailers gain less than with non-cooperation. This also implies that the total profit of partially-cooperating retailers may decrease with respect to non-cooperation.*

This result is interesting and even surprising because usually, in game theory, players are better off when they cooperate.

This result is not intuitive for symmetric players, i.e. symmetric model, and for an asymmetric model. Indeed, even if we could get an asymmetric solution, where one player gains from partial-cooperation and another looses, usually the total gain of players improves with partial-cooperation. This supposes a bargaining mechanism of sharing the gains from partial-cooperation between retailers, particularly when there is a looser and a winner. Therefore, our result is new and interesting because it shows that the profit of each retailer may decrease with partial-cooperation with respect to non-cooperation. Because of symmetry of both the model and the solution, the total profit of retailers may decrease with partial-cooperation.

Our principal finding is due to the spending in national advertising. Indeed, when retailers unilaterally cooperate, the retail price increases and the spending in national advertising decreases, leading to an important decrease in sales and therefore in retailers' profits. With the present model, we can easily show that, when consumers' demand is not affected by advertising, the retailers' profits are always higher under partial-cooperation.

Let's notice that when retailers are symmetric, Yang and Zhou (2006, Proposition 2) showed that each partially-cooperating retailer gains more than with non-cooperation. However, when retailers are asymmetric and face different demand functions, one retailer may be worse off, whereas the other may be better off, with partial-cooperation (Yang and Zhou, 2006, page 112, inference (4) and Table 3). They have not given any response to the total profit of retailers. We have verified that the total profit of retailers is always higher with partial- cooperation than without cooperation, by using their Table 3. All their results are intuitive. Therefore, our principal result, stating that each retailer's profit and the total profit of retailers may be higher with non-cooperation, constitutes a non-obvious result. It is a new and interesting result when compared to the findings of Yang and Zhou (2006).

Wang *et al.* (2011, Theorem 4) showed that partially-cooperating retailers may be worse off under some conditions for the symmetric special case and the Stackelberg game. Nonetheless, their findings are obtained under heavy hypotheses: the marginal profits per unit of retailers and the manufacturer, and consequently the selling prices of retailers, are exogenously determined. This implies that these latter are the same whether retailers partially-cooperate or not, making retailers' profits comparisons between Stackelberg non-cooperative and partially-cooperative cases questionable.

Proposition 5. $(i)\, \pi_r^* < \bar{\pi}_r \Leftrightarrow 3-\sqrt{5} < \beta < 1, (ii)\, \bar{\pi}_m < \pi_m^*, (iii)\, \bar{\pi}_t < \pi_t^* < \pi_t^c.$

When the degree of substituability between the two products is sufficiently high, we have the standard result that partial-cooperation of retailers increases their profits with respect to non-cooperation.

Also, partial-cooperation decreases the manufacturer's profit because it does not modify the wholesale price while decreasing the quantities sold. Our computations show that, even when the profits of retailers increase with partial-cooperation, the total profit of the supply channel always decreases with respect to non-cooperation. Finally, and as expected, the total profit of the supply channel is the highest with full-cooperation.

Let's notice that, for the manufacturer's and the whole channel's profits, Yang and Zhou (2006, Propositions 4 and 5, Table 3) found similar results as ours, whereas Wang *et al.* (2011, Theorem 4) showed that partial-cooperation may be harmful or good for the manufacturer and the whole channel.

Proposition 6. $(i)\, \overline{CS}_t < CS_t^* < CS_t^c, (ii)\, \bar{S} < S^* < S^c.$

The above Proposition shows that cooperation between retailers reduces consumers' surplus. This result is not as obvious as one may think. Indeed, our expression (11) shows that consumers' surplus is not positively dependent only on price-demand, as usual in micro-economic theory. But it depends positively also on the spending in advertising, which is a new consumers' surplus evaluation that we propose. Since partial-cooperation increases retail prices, leading to a decrease in price-demands, and decreases the spending in advertising, consumers' surplus, i.e. consumers' welfare, decreases.

However, full-cooperation reduces retail prices and increases advertising spending, leading to an increase in consumers' welfare. By taking into account the total profit of the supply channel, we can conclude that the worst situation for consumers and the social welfare is the partial-cooperation case, and the better one is the full-cooperation case. Let's remind that previous studies have not evaluated the impact of cooperation between retailers or between all members of the supply channel on consumers and social welfare.

8. Conclusion

Our paper extends the growing literature on supply channel by considering a Stackelberg manufacturer–two-retailers relationship. We evaluate the impact of cooperation between retailers and between all channel members on profits, consumers and social welfare.

The manufacturer produces one product that he sells to the two retailers. These latter sell only the manufacturer's products to consumers. The manufacturer decides on the wholesale price and uses brand advertising to attract consumers and to increase the effective-demand for the product. Retailers decide on the retail prices. Consumers' effective-demand for the product depends on the retail prices of the two retailers and on the manufacturer's advertising spending.

First, we model the decision process as a non-cooperative game in which the manufacturer is the leader and the two competing retailers are followers. The manufacturer chooses the spending in national advertising and the wholesale price, and then each retailer chooses its price to consumers. Then, we consider the partial-cooperation case where retailers maximize a joint profit function. In the full-cooperation case, all members of the supply channel maximize the total channel profit.

We show that the wholesale price does not depend on whether retailers cooperate or not. With partial-cooperation, the retail price is the highest, whereas the quantity purchased and expenditures in advertising are the lowest.

When the degree of substituability between the two products proposed by retailers is sufficiently low, both cooperating retailers gain less than with non-cooperation. This result is interesting and even surprising because usually firms are better off when they cooperate. This result is due to the spending in national advertising. Indeed, when retailers unilaterally cooperate, we showed that this increases the retail prices and reduces the spending in national advertising leading to an important decrease in sales and in retailers' profits. When the degree of substituability between the two products is sufficiently high, we have the standard result that partial-cooperation of retailers increases their profits with respect to non-cooperation. In addition, cooperation between retailers decreases the profit of the manufacturer because there is no change in the wholesale price and the quantities sold are diminished. As a result, and independently of whether the degree of substituability between the two products is high or low, the total profit of the supply channel is the lowest with partial-cooperation.

As expected, full-cooperation gives the highest total profit for the supply channel. Channel members can share the extra-profit due to full-cooperation by setting a wholesale price which is lower than those of non-cooperation and partial-cooperation. There exists a cooperative wholesale price that splits the extra-profit equally between the manufacturer and the two retailers. We propose a cooperative implementable contract between all channel members.

We propose a new and unusual evaluation of consumers' surplus which does not depend only on the price-demand function, but it also depends positively on the spending in national advertising. We show that the worst situation for consumers and the social welfare is partial-cooperation, and the better situation is full-cooperation.

We agree that our model is simple and tractable. As most of papers, we use a special form of sale response function. We think that many of our results can be generalized to other forms of demand functions for goods, especially when these latter are not linear.

Finally, this model can be extended by considering that retailers spend in local advertising, and that the manufacturer pays a fraction of this local advertising cost. Such an extension complicates enormously the tractability of the model and necessitates numerical methods. We think that the introduction of local advertising will not change our results when the impact on effective-demand and on firms' profits of this latter is weak compared to the impact of national advertising.

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