

The main objective of the present paper is to present a theoretical model, based on the expectations-adjusted Phillips Curve and on Okun's Law, in which, given a country and its generic business partner, the difference between effective inflation and expected inflation in a given country is related to the effects of the economic cycle on the GDP of the national economy and of the economy of the partner considered.

In this model, finalized not only to explain the effects of the economic cycle on the GDP growth rate but also useful for forecasting purposes, the authors present a linear relationship that links the difference between the effective inflation rate and the expected inflation rate (inflation gap) of the national economy, with the difference between the cosine of the real GDP growth rate of the national economy and the cosine of the growth rate of the real GDP of the trading partner. This last difference represents the effect, on the inflation gap, of the economic cycle in the national economy and in the economy of the considered commercial partner; the inflation gap, by effect of the economic cycle in the two economies involved, follows, as the authors will later highlight, the trend of a periodic wave. A positive or negative change in inflation can lead to a positive or negative change, in the difference in growth between the national economy and the foreign economy, due to the effects of the economic cycle.

The model was obtained using the simplest formulations of Friedmann's expectations-augmented Phillips Curve and of Okun's law and assuming that real exports and real imports of Germany are a linear function of real GDP (national and foreign), of the real exchange rate (RER), of the final consumption expenditure in real terms of private operators and of the government (in the national and foreign economy), of the index of export and import prices (export price index and import price index) and of sine of the real growth rate of national GDP and of foreign GDP. It has also been assumed (and verified) that the expected inflation at time $t + 1$ is equal to the cyclical component of the effective inflation rate at time t , namely, that the operators formulate their expectations giving greater weight to the assets characterized by greater volatility of price.

In the authors' opinion, this simple theoretical model, appropriately verified empirically, represents a useful predictive tool for the effects of the economic cycle on the real GDP of the national economy and of the economy of the business partner in question. In other words, the results, obtained from the econometric verification, demonstrate not only the existence and the significance of the relationship sanctioned by the theoretical model and the validity of the macroeconomic assumptions on which it is built, but also the ability of the latter to predict with good precision the effect of the economic cycle on the real GDP growth rate in the countries of the euro area, given the inflation gap in Germany.

The econometric analyses, carried out in the paper, also seem to suggest that economic operators, in a given economy, can formulate their own expectations about future inflation using the cyclical component of the effective inflation rate by Consumer Price Index, namely the short-term changes due mainly to goods that, according to economic theory (Taghizadeh-Hesary et al.2018), traditionally have a greater price volatility, such as the electricity and the foodstuffs.

The model used appears statistically robust and internally valid, besides to present an innovative character; in fact, the theme treated by the authors, in this article, seems not to have been the object of study by other authors until now; that is, currently there are no studies that relate the effects of the economic cycle to the expected inflation rate. Furthermore, the ability of inflation, within a given economy, to determine and to predict the effects of the economic cycle on the real GDP of the national economy and of its trading partners constitutes undoubtedly an interesting contribution even with respect to the theory of international trade.

The paper is organized as follows.

In section 1, after the introduction, are analysed the research aims while in section 2 the instructions and procedures experimental used; in section 3, instead, the theoretical model and the relative estimation methods are introduced. In section 4 the results of the model estimation are reported and some diagnostic tests related to them are examined; subsequently, it is carried out, through an ARMAX model, a static forecast of the GDP growth rate. Section 5, finally, provides a conclusion and contains a general discussion on directions for future research.

The authors hope, furthermore, that the paper is a very useful for stimulating research at the interface of mathematics and economics (useful, namely, in bridging economics and mathematics, in particular) and that it builds a link that can become advantageous for both disciplines involved.

Instructions and procedures experimental used

In addition to what is specified in the research objectives, the authors add the following informations on the instructions and procedures used in the research.

The estimation of the model parameters and their validation were carried out with historical series data on a quarterly basis and with type models: multivariate linear regression (OLS) and Autoregressive Moving Average with exogenous inputs (ARMAX). In particular, the procedure adopted in the paper consists in using the vector regression models (VAR) by means of which the authors have tried to expose an econometric analysis to study the effects of the inflation gap on the GDP growth rate, due to business cycle effects. The economies examined for econometric verification are Germany and the euro area countries, considered, in their complex, like a single economy. The sample period of the data ranges from the first quarter of 1999 to the first quarter of 2018.

The historical series relating to the final consumption expenditure of private operators and of the government in Germany, to the final consumption expenditure of private operators and of the government in the euro area, to the German export price index and to the German import price index, were collected through the software Datastream of Thomson Reuters. All other variables come from the online database of the Federal Reserve Bank of Saint Louis

(FRED). The two variables *hpt_GrowthEur* and *hpt_GrowthGer* (in models 1 and 2 the paragraph 3) were obtained applying a Hodrick-Prescott filter with $\lambda = 1600$, respectively to the real GDP growth rate of the euro area and to the real GDP growth rate of Germany. The variables *RERGer* and *REREur* have been computed by the authors using the nominal exchange rate, listed certain to uncertain, between Germany and the euro area (which, of course, is fixed) and the Consumer Price Index of both economies.

In model 3 (of the paragraph 3) the exogenous variable *CyclicalInfGert1* was obtained applying a Hodrick-Prescott filter with $\lambda = 1600$ to the effective German inflation rate (calculated at the German CPI) and was modeled according to an AR(1) process. The statistical software used for all the elaborations is Gretl.