International Journal of Economics and Financial Issues

ISSN: 2146-4138

available at http: www.econjournals.com

International Journal of Economics and Financial Issues, 2023, 13(1), 7-12.

Valuation of Corporate Debt and Equity in Uncertain Markets

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Received: 20 September 2022 **Accepted:** 13 December 2022 **DOI:** https://doi.org/10.32479/ijefi.13706

ABSTRACT

In practice, financial decisions are made in the context of indeterminacy. Randomness, uncertainty, and fuzziness are three basic types of indeterminacy. A multiplicity of differential equations have been designed to depict various processes powered by different kinds of indeterminacy. Among others, these differential equations include uncertain differential equations, stochastic differential equations, and fuzzy differential equations. In this study, we propose that the value of a firm can be described by an uncertain differential equation powered by a geometric canonical Liu process. Uncertain differential equations describe processes driven by uncertainty. Implementing the uncertain Liu option pricing theory, we develop and analyse a framework for valuing debt and equity for a levered firm in uncertain markets. Numerical calculations are demonstrated.

Keywords: Uncertainty Theory, Uncertain Liu Option Pricing Theory, Uncertain Markets, Corporate Debt, Corporate Equity
JEL Classifications: G12, G13

1. INTRODUCTION

Decisions, in reality, are executed in the context of indeterminacy (Chen et al., 2022; Huang and Ning, 2021; Matenda and Chikodza, 2019). The indeterminacy of event outcomes is their quality or situation of being unpredictable in advance. Randomness, uncertainty, and fuzziness are three rudimentary types of indeterminacy. Matenda and Chikodza (2019) opined that randomness is a feature of anything that probabilistic laws can explain, and uncertainty is a characteristic of anything that can be explained by belief degrees. Fuzziness refers to any phenomenon that a credibility measure can quantify (Matenda, 2016). Kolmogorov (1933) introduced probability theory to describe randomness, and Liu and Liu (2002) introduced credibility theory to deal with fuzziness effectively. In 2007, Liu (2007) suggested uncertainty theory to model uncertainty.

Probability theory is a subdivision of pure mathematics that deals with frequencies. It is implemented when the size of the sample is relatively large to determine the probability distribution from obtainable frequency (Gao et al., 2022; Zhou et al., 2022; Huang and Ning, 2021; Liu et al., 2022; Wang and Ralescu, 2021; Matenda and Chikodza, 2019). The implementation of probability theory in finance theory resulted in the emergence of stochastic finance theory. That is to say, stochastic finance theory is founded on probability theory. Stochastic processes (e.g., the Brownian motion) and stochastic differential equations (DEs) are indispensable tools in probability theory and stochastic finance theory. A stochastic process is defined as a series of random variables indexed by space or time. Stochastic DEs are DEs powered by the Brownian motion. On the other hand, credibility theory is a unit of mathematics that deals with and examines the aspects of dynamic fuzzy phenomena. Credibility theory deals with events or processes whose measurement is imperfect and dim (see, for instance, Jiwo and Chikodza, 2015). The application of fuzzy mathematics in finance theory resulted in fuzzy finance theory. That is to say, fuzzy finance theory is founded on fuzzy mathematics. Fuzzy processes (e.g., the geometric Liu process) and fuzzy DEs are essential tools in fuzzy calculus and fuzzy finance theory. A fuzzy process is defined as a series of fuzzy variables which change with time. Fuzzy DEs are DEs powered by the Liu process.

Derivative values depend on the values of the underlying assets. Options are one prominent example of derivative instruments and are widely used in risk management. An option is defined as a contract that offers the holder of the contract the privilege but not the compulsion to sell or buy an underlying asset at a stated fixed price on a specific date or at any time before that date. Options theory is a vital discipline in modern finance and it is essential in both the financial industry and academia (Gao et al., 2022; Huang and Ning, 2021; Hassanzadeh and Mehrdoust, 2018). Wang and Ralescu (2021) and Hassanzadeh and Mehrdoust (2018) articulated that the valuation of options is a challenge in finance. In the same vein, Rao and Zhu (2022) postulated that the major challenge in stock option pricing is discovering an appropriate process to explain the stock price movement patterns better. In the theory of stochastic analysis, Black and Scholes (1973), premised on the geometric Brownian motion, introduced the well-known Black-Scholes (B-S) model and proposed European option pricing formulae. Since then, the B-S model has become an essential instrument in stochastic finance theory when pricing financial derivatives (Wang and Ralescu, 2021; Peng and Yao, 2011). The classical work of Black and Scholes (1973) has provided a strong footing for derivative instruments pricing. On the other hand, Liu (2008) suggested a fuzzy stock model called Liu's stock model. The Liu's stock model is considered a fuzzy counterpart of the B-S model and has represented a significant advancement in fuzzy finance theory. After that, Qin and Li (2008) deduced the conforming European option pricing formulae.

It is important to note that most studies on option pricing have been performed under the frameworks of fuzzy DEs and stochastic DEs. Nonetheless, in some instances, some inexact quantities (e.g., stock prices and corporate values) denoted by human language do not act like fuzziness or randomness (Rao and Zhu, 2022; Huang and Ning, 2021; Hassanzadeh and Mehrdoust, 2018). To deal with such inexact quantities, domain specialists are summoned to examine their belief degrees of every event happening (Gao et al., 2022; Huang and Ning, 2021; Liu et al., 2022; Wang and Ralescu, 2021; Matenda and Chikodza, 2019; Hassanzadeh and Mehrdoust, 2018). The authors (Matenda and Chikodza, 2019) further articulated that credibility theory or probability theory cannot describe personal belief degrees because it leads to counterintuitive results. Therefore, to rationally model personal belief degrees, Liu (2007) developed the uncertainty theory that was further analysed by Liu (2010). Liu (2015) defined the uncertainty theory as the subdivision of pure mathematics that models belief degrees. The implementation of the uncertainty theory in finance theory led to the emergence of uncertain finance theory. That is to say, the uncertain finance theory is founded on uncertainty theory. Uncertain processes (e.g., the canonical Liu process) and uncertain DEs play critical roles in the uncertainty theory and uncertain finance theory (see, for instance, Chen et al., 2022; Wang and Ralescu, 2021). An uncertain process refers to a series of uncertain variables indexed by space or time. Uncertain DEs are DEs powered by the canonical Liu process.

Under the supposition that stock prices obey the geometric canonical Liu process, Liu (2009) derived an uncertain stock model, i.e., the uncertain Liu stock model, and developed the conforming European option pricing formulae. From that time, the uncertain Liu stock model has emerged as an essential instrument in uncertain finance theory. The uncertain Liu stock model is generally regarded as the uncertain equivalent of the B-S model and Liu's stock model. Sun and Chen (2015) and Chen (2011) generated the Asian option pricing formulae and American option pricing formulae, respectively, premised on the uncertain Liu stock model. Also, premised on the uncertain Liu stock model, Chen et al. (2013) and Peng and Yao (2011) designed a periodic-dividend stock model and a mean-reverting stock model, respectively. Further, Yao (2015) developed a sufficient and necessary no-arbitrage condition for the uncertain Liu stock model.

The total market value of the corporate is given by the total market value of its outstanding equity plus the total market value of its outstanding debt and other external claims. Conceptually, equity in a levered corporate resembles a call option on the corporate. The selling of pure discount bonds by shareholders to the bondholders is virtually the same as the selling of corporate's assets to the bondholders for the bond issue's proceeds and buying a call option to reacquire those corporate assets from the bondholders at a strike price equivalent to the issued bonds' face value (see Black and Scholes, 1973). Further, shareholders, as the owners of the corporate, are residual claimants. They claim the remaining cash flows after all financial claimants are paid. Likewise, if an organisation is liquidated, equity-holders get the residual after all financial claimants are paid. In publicly traded organisations, the basic principle of limited liability protects shareholders. If the corporate value is lower than the outstanding debt value, losses incurred by the shareholders cannot exceed their total investment into the organisation. When the debt value exceeds the value of the assets, equity-holders can move away from the firm. On the contrary, when the value of the assets is more than the value of the debts, the equity-holders would continue operating the business.

Corporate liabilities have been widely priced under the framework of stochastic DEs (Eissa and Elsayed, 2022; Yin et al., 2018; Black and Scholes, 1973). Black and Scholes (1973) illustrated how the B-S model can be used in the pricing of corporate liabilities. Nevertheless, studies devoted to the valuation of corporate liabilities under the frameworks of fuzzy DEs and uncertain DEs are limited. In this study, we propose that the corporate value can be modelled by an uncertain differential equation powered by the geometric canonical Liu process. The article implements the uncertain Liu option pricing theory to develop and analyse a framework for valuing debt and equity for a levered firm in uncertain markets. Numerical calculations are demonstrated. We indicate that market participants can apply this framework in practice to value a levered firm's corporate debt and equity in uncertain markets. To the authors' knowledge, no such study has been conducted before.

The remaining part of the study is categorised as below. Section 2 describes the preliminaries, and Section 3 gives an overview of the uncertain Liu stock model. The uncertain Liu option pricing model is presented in Section 4 and the dynamics of valuing debt and equity for a levered firm in uncertain markets are illustrated in Sections 5. Section 6 concludes the study.

2. PRELIMINARIES

Here, we present some essential definitions in uncertainty theory. These definitions constitute the footing of this article. In this study, we take into account an uncertainty space (Γ, Ł, Ɱ) and a filtration, $\{\mathscr{F}_t\}_{t\geq 0}$, produced by a one-dimensional canonical Liu process, $\{C_t\}_{t\geq 0}$.

Definition 2.1 (Liu, 2007) Assume that Γ is a non-empty set and Ł is a σ-algebra over Γ. Each element Λ in Ł is considered to be an event. An uncertain measure refers to a set function Ɱ: Ł → [0, 1] which meets the below-mentioned four axioms:

- Normality Axiom: Ɱ{Γ} = 1;
- Monotonicity Axiom: $Ɱ\{\Lambda_1\} \leq Ɱ\{\Lambda_2\}$ if $\Lambda_1 \subset \Lambda_2$;
- Duality Axiom: $Ɱ\{\Lambda\} + Ɱ\{\Lambda^c\} = 1$ for every Λ ∈ Ł;
- Sub-additivity Axiom: For each countable event sequence $\{\Lambda_1, \Lambda_2, \ldots\}$, $Ɱ\{\cup_i \Lambda_i\} \leq \Sigma_i Ɱ\{\Lambda_i\}$.

Definition 2.2 (Liu, 2007) An uncertain variable is a measurable function ξ from an uncertainty space (Γ, Ł, Ɱ) to the set of real numbers such that for each Borel set B, {ξ ∈ B} is an event.

Definition 2.3 (Liu, 2013) Assume that T is an index set and (Γ, Ł, Ɱ) refers to an uncertainty space. An uncertain process is a measurable function X_t (γ) from T x (Γ, Ł, Ɱ) to the set of real numbers such that at any time t, for every Borel set B, $\{X_t \in B\}$ is an event.

Definition 2.4 (Liu, 2009) An uncertain process C_t is defined as a canonical Liu process if

- $C_0 = 0$ and almost all sample paths are Lipschitz continuous,
- C_t has independent and stationary increments,
- each increment C_{s+t} - C_s is a normal uncertain variable with variance t^2 and expected value 0, whose uncertainty distribution is given by

$$\Phi(x) = (1 + \exp(\frac{-\pi x}{\sqrt{3t}}))^{-1},\ x \in \mathcal{R}. \tag{1}$$

Definition 2.5 (Haugh, 2010) A filtration, $\{\mathscr{F}_t\}_{t\geq 0}$, describes the time evolution of information. A filtration on (Γ, Ł, Ɱ) is an index set, $\{\mathscr{F}_t\}_{t\geq 0}$, of sub-σ-algebra of $\mathscr{F}$.

Definition 2.6 (Liu, 2008) Assume that C_t is a canonical Liu process, and that f and g are two functions. The equation

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t \tag{2}$$

is an uncertain DE. A solution is an uncertain process X_t that satisfies (2) identically in t.

Remark 2.1: An uncertain DE (2) is considered to be equivalent to an uncertain integral equation

$$X_s = X_0 + \int_0^s f(t, X_t)dt + \int_0^s g(t, X_t)dC_t. \tag{3}$$

3. UNCERTAIN LIU STOCK MODEL

Liu (2009) proposed that the price of the stock can be described by an uncertain DE driven by the geometric canonical Liu process and derived an uncertain Liu stock model in which the price of the bond D_t and the price of stock Y_t are given by

$$dD_t = rD_t dt \tag{4}$$

and

$$dY_t = \mu Y_t dt + \sigma Y_t dC_t, \tag{5}$$

respectively, where r denotes the risk-free rate of interest, μ is the log-drift, σ represents the log-diffusion, and C_t denotes the canonical Liu process. The explicit price of the bond is described by

$$D_t = D_0 \exp_{(rt),} \tag{6}$$

and that of the stock is given by

$$Y_t = Y_0 \exp(\mu t + \sigma C_t), \tag{7}$$

whose inverse uncertainty distribution is

$$\phi_t^{(-1)}(\alpha) = Y_0 \exp(\mu t + (\frac{\sigma t\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}). \tag{8}$$

4. UNCERTAIN LIU OPTION PRICING MODEL

Liu (2009) suggested the following formulae for the European put option premium, p, and the European call option premium, c. Suppose the European call option for the uncertain Liu stock model is associated with an exercise price K and expiration time t. The European call option premium is then described by

$$f_c = \exp(-rt)\int_0^1 \left(Y_0 \exp\left(\mu t + \frac{\sigma t\sqrt{3}}{\pi} \ln\left(\frac{\alpha}{1-\alpha}\right)\right) - K\right)^+ d\alpha. \tag{9}$$

Assume that the European put option for the uncertain Liu stock model is associated with an exercise price K and expiration time t. The European put option premium is then given by

$$f_p = \exp(-rt)\int_0^1 \left(K - Y_0 \exp\left(\mu t + \frac{\sigma t\sqrt{3}}{\pi} \ln\left(\frac{\alpha}{1-\alpha}\right)\right)\right)^+ d\alpha\ . \tag{10}$$

5. PRICING CORPORATE DEBT AND EQUITY IN UNCERTAIN MARKETS

The total market value of the corporate is given by the total market value of outstanding equity plus the total market value of outstanding debt and other external claims. Since the equity in a levered corporation resembles a call option on the corporate, we will use the uncertain Liu option pricing theory to price the debt and equity of a levered firm in uncertain markets. Here, we assume the following:

- Pure discount bonds: The corporate of interest sells pure discount bonds, which forbid any coupon payments until after their maturity, and the residual is paid-off to equity- holders.
- Capital structure cannot affect the total value of the firm.
- Investors have uniform anticipations concerning the dynamic behaviour of the corporate's assets.

In mathematical notation, from the balance sheet, the corporate's value is given by

$$V = E + D, \tag{11}$$

where V represents the total market value of the corporate, E denotes the total market value of equity outstanding, and D represents the total market value of the outstanding debt and other external claims.

Let's say B is the face value, i.e., par value, of defaultable debt. Therefore, the pay-offs of corporate debt and equity at debt maturity are given by

Total debt: $D = \min(B, V)$,

and

Total equity: $E^* = \max(V - B, 0)$.

Similarly, the value of the call option c, given an exercise price K, on an asset with a present-day value S, is described by

$c = \max(S - K, 0)$,

and its pay-off is given by

$c = \exp(-rt) \max(S - K, 0)^+$.

The dynamics of equity and call option pay-offs are similar. This confirms that the equity of a firm resembles a call option. Therefore, we propose that the uncertain Liu option pricing model might offer a modelling mechanism that can be used in the valuation of equity and debt for a levered corporate.

The total corporate value, V_t, at time t is given by the following uncertain DE powered by the geometric canonical Liu process

$$dV_t = \mu V_t dt + \sigma V_t dC_t, \tag{12}$$

where μ represents the log-drift, σ denotes the log-diffusion, and C_t is the geometric canonical Liu process.

Theorem 1. *Let E* be the equity value, B be the bond's face value, V be the corporate's value, ∝ be the log-drift, σ and be the log-diffusion. The value of the corporate equity is given by*

$$E^* = \exp(-rt)\int_0^1 \left(V_0 \exp\left(\mu t + \frac{\sigma t\sqrt{3}}{\pi} \ln\left(\frac{\alpha}{1-\alpha}\right)\right) - B\right)^+ d\alpha . \tag{13}$$

Proof: The solution to equation (12) is an uncertain process given V_t by

$$V_t = V_0 \exp(\mu t + \sigma C_t) \tag{14}$$

and its inverse uncertainty distribution is described by

$$\psi_\alpha^{-1} = V_0 \exp(\mu t + \sigma \Phi_\alpha^{-1}) \tag{15}$$

Now the expected value of the pay-off for total equity E* is given by

$$E^* = \exp(-rt)\, E(\max(V_t - B, 0+)). \tag{16}$$

The uncertainty distribution of equation (16) is described by

$$\Phi_t^* = \mathcal{M}\{\exp(-rt)(V_0 \exp(\mu t+\sigma C_t)-K)\le x\}$$

$$= \mathcal{M}\left\{\left(\exp\left(\mu t + \sigma C_t\right)\right) \le \frac{K + x\exp(rt)}{V_0}\right\}$$

$$= \mathcal{M}\left\{C_t \le \frac{1}{\sigma}\ln\frac{K + x\exp(rt)}{V_0} - \frac{\mu t}{\sigma}\right\}$$

$$= \left(1 + \frac{\pi\mu}{\sqrt{3}\sigma} + \frac{\pi}{\sqrt{3}\sigma t}\ln\frac{V_0}{K + x\exp(rt)}\right)^{-1}$$

whose inverse uncertainty distribution is

$$\psi^{*-1}_{\alpha} = \exp(-rt)\, V_0\, \exp\left(\mu t + \frac{\sigma t\sqrt{3}}{\pi}\ln\left(\frac{\alpha}{1-\alpha}\right)\right) - B$$

Applying the expected value of an uncertain variable definition to equation (16), we get

$$E^* = \exp(-rt)\int_0^1 \left(V_0 \exp\left(\mu t + \frac{\sigma t\sqrt{3}}{\pi} \ln\left(\frac{\alpha}{1-\alpha}\right)\right) - B\right)^+ d\alpha$$

which is similar to equation (13). Thus, the proof is concluded.

Further, if V is the value of the firm and E* is the value of the outstanding equity, the value of the outstanding debt, D, is given by D = V–E*, i.e.,

$$D = V - (\exp(-rt))\int_0^1 \left(V_0 \exp\left(\mu t + \frac{\sigma t\sqrt{3}}{\pi} \ln\left(\frac{\alpha}{1-\alpha}\right)\right) - B\right)^+ d\alpha. \tag{17}$$

In general, the value of the debt can be presented by

$D = D(V, B, t, \sigma, r)$,

where $\frac{\partial D}{\partial V}, \frac{\partial D}{\partial B} > 0$ and $\frac{\partial D}{\partial t}, \frac{\partial D}{\partial \tilde{A}}, \frac{\partial D}{\partial r} < 0$. These partial effects have the following intuitive interpretations:

- If the firm value increases, equity value and cover on the debt increase. Consequently, the belief degree of default decreases while the debt value rises

- If the debt repayment amount increases, the claims of debt-holders against the assets of the firm increase. Hence, debt value increases, and the current equity value decreases.
- The present debt value decreases and the equity market value increases when the debt repayment increases.
- An increase in the riskless interest rate lowers the debt's present value and amplifies the equity market value.
- The dispersion of possible values of the corporate value at the debt maturity date increases when the time to maturity or the log-diffusion increases.
- The debt-holders receive B as their maximum payment. When the dispersion of possible outcomes of B increases, the belief degree that the corporate assets' value will be below B increases. Thus, the belief degree of default increases, the debt value decreases, and the equity value increases.

5.1. Numerical Example: Valuation of Corporate Debt and Equity

Let the corporate value V be \$100 million, the outstanding debt's face value B be \$80 million, the life of a zero-coupon debt t be 2 years, the log-diffusion σ be 0.32, the rate of treasury bond commensurate to the life of the option be 0.08, and the log-drift μ be 0.06. Given the above variables, using the uncertain Liu option pricing theory, the equity value E^* = \$50.764 million, and the value of outstanding debt D = \$49.236 million.

Considering the above variables, let's look at firm value dynamics and equity pay-off dynamics. Figure 1 shows the relationship between the firm's value, time, and log-diffusion. It is clear from Figure 1 that if all other parameters are held constant, the firm value increases exponentially when time and log-diffusion increase.

In Figure 2, the firm value increases exponentially as the time and log-drift increase.

From Figures 1 and 2, we deduce that increases in the firm value due to rises in the value of the log-drift for a fixed value of log-diffusion are greater than those due to rises in the log-diffusion value for a fixed value of log-drift over the same time interval.

The relationship between the equity pay-off, time, and the risk-less interest rate is shown in Figure 3, and it is clear that there is an exponential relationship between these parameters. The equity pay-off increases exponentially as time and the risk-less interest rate increase.

Figure 4 shows the relationship between the equity pay-off, debt's face value, and time. The equity pay-off decreases linearly as the debt's face value increases and increases exponentially as time increases.

In this study, we proposed that the value of a firm can be described by an uncertain DE powered by the geometric canonical Liu process. Applying the uncertain Liu option pricing theory, we developed and analyse a framework for valuing debt and equity for a levered firm in uncertain markets. We provided numerical calculations for the designed model. Numerical computations indicated that debt and equity for a levered firm can be priced in uncertain markets. This study could be extended by valuing (i) equity in a troubled corporate; (ii) the effect of a conflict between bondholders and shareholders on the corporate value; (iii) a corporate after stockholders pursued a project; (iv) equity after a conglomerate merger; and (v) a new corporate.

Figure 1: Relationship between the value of the firm, time and log-diffusion

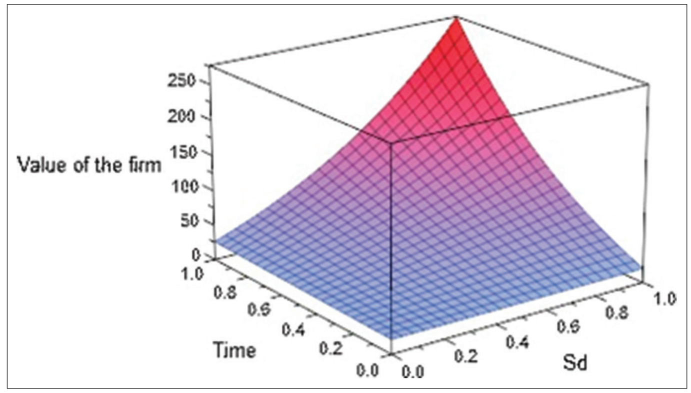


Figure 2: Relationship between the value of the firm, time and log-drift

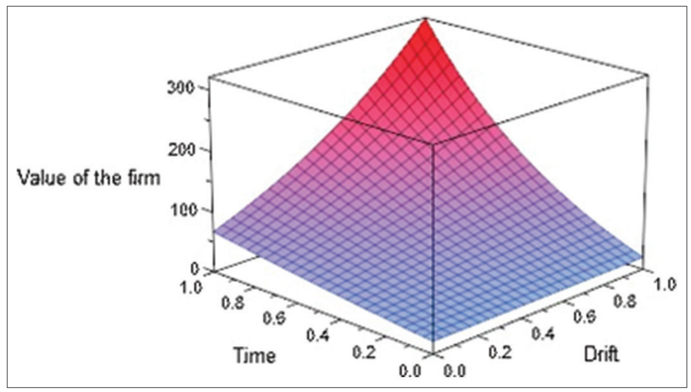


Figure 3: Relationship between the equity pay-off, time, and risk-less interest rate

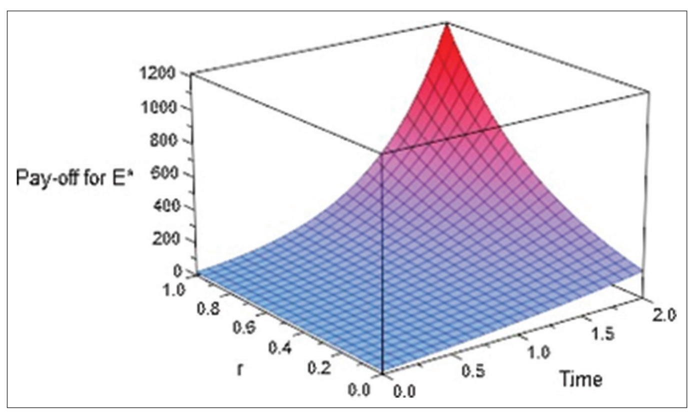
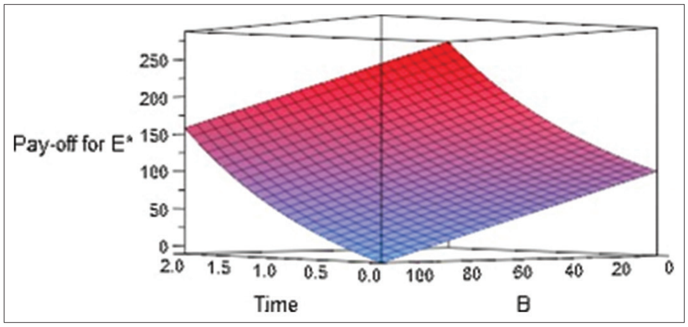


Figure 4: Relationship between equity pay-off, time, and debt's face value given a fixed value of log-diffusion



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