The Interaction of Major Crypto-assets, Clean Energy, and Technology Indices in Diversified Portfolios

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ABSTRACT

Cryptocurrencies have gained high interest from media, regulatory authorities, and both retailer and institutional investors especially in the COVID-19 pandemic followed by a tremendous academic interest. Classification of cryptos as an asset is one important issue while another crucial topic is the significantly high return and volatility fluctuations which does not make life easier for portfolio investors. High energy consumption to produce cryptos and carbon emission issues are also not developing the favor of cryptocurrencies. Moreover, in financial literature recent studies show that prices of renewable energy stocks have long-term cointegrating relationship with technology companies. In this context first we employ an asymmetric VAR-GARCH model to study spillover effects between major crypto-currencies, clean energy, and technology indices. Using daily data of the two major cryptocurrencies for the period of January 01, 2016 and September 30, 2021, we relate risk and return of different mean-variance portfolio strategies to Bitcoin (BTC), Etherium (ETH), S&P Global Clean Energy Index (SPGCE) and MSCI World Information Technology Index (MSCIWIT). Secondly, we apply the Markowitz mean-variance framework to assess risk-return benefits of cryptocurrency-portfolios. Our main goal is to offer optimal portfolio allocation approaches including cyptocurrencies with traditional financial assets. We will combine cryptocurrencies, clean energy, and technology indices to maximize return and Sharpe ratio. Furthermore, we will use our asymmetric VAR-GARCH models results to understand and cross check the Markowitz portfolio allocation results in details.

Keywords: Cryptocurrencies, Portfolio Optimization, Markowitz, Spillover, VAR-VECH-TARCH
JEL Classifications: G11, C58, G14

1. INTRODUCTION

In recent years, the growing appetite for risk in the markets, coupled with increased liquidity globally, increased demand for crypto assets. The high volatility increased the risk desire of many investors, while many other were hurt by high volatility. Bitcoin fell to $ 3045 after reaching the level of $19,783 in December 2017, an 85% depreciation. Consequently, Bitcoin broke this historic peak, which was tested in 2017, back in November 2020. During this period markets were bearish and many investors had to clear their positions with losses. Moreover, the period of abundant liquidity encouraged investors to find new investment areas to achieve higher profits. A high inflation era both in developed and

developing economies also occurred at the end of 2021 which brought a harder challenge for portfolio managers, especially alpha investors, to achieve returns higher than inflation.

In this context, traditional financial assets challenged by high inflation rates and fueled by digitalization in the finance industry, crypto assets are mentioned more frequently by both individual and institutional investors. Although they were strongly opposing to crypto assets before, largest banks such as JP Morgan and Goldman Sachs could not stand against crypto invasion and started to offer cryptocurrency services to asset customers. Hence, it is not hard to guess that number of such banks and funds providing crypto services will increase in the coming processes. High level

of interest for crypto assets especially during the pandemic period, many institutions have started providing crypto services, while central banks have accelerated their digital currency work in the face of increasing demand. Investopedia[1] reported that bitcoin surged 90 per cent during the pandemic.

In the guide prepared by IOSCO for investors, crypto assets are defined with different functions and features.[2] According to this guidance, crypto assets can be classified into different categories, such as financial instruments, securities, commodities or payment instruments. Hence, growing institutional adoption and increased inflationary persistency have lifted cryptocurrencies to all-time highs during the pandemic, but not without too high volatility. Just after Tesla carried Bitcoin to new price highs by announcing its $1.5 billion investment, prices crashed nearly 20% when Elon Musk announced prices seemed "a little high" via Twitter. Moreover, these assets are already included in the technology company's portfolios. As of September 2020, the ratio of crypto assets in the portfolios of technology companies such as MacroStrategy LLC, Tesla, Square is increasing day by day. Crypto assets account for 85 percent of MacroStrategy's total assets, and bitcoin alone accounts for 54 percent of this ratio. Tesla has a crypto asset portfolio of 0.3 percent of its total assets.

It is clear that the demand of corporate investors, as well as individual investors and mutual funds increased to crypto assets (Tables 1 and 2). Boosted with digitalization effect, crypto assets became a major competitor of investment instruments such as stocks, fixed income securities, commodities, gradually increasing their share in the traditional financial system. Supply limitation cryptocurrencies also protects the value of it against inflation and emerge as an investment alternative in the market. Looking at the performance of Bitcoin and Ethereum over the past year, we see that the returns on these assets are much higher compared to other investment instruments.

Another aspect of cryptocurrencies is its mutual relationship with energy, especially renewables, companies. Many energy companies started to invest in cryptocurrencies suggesting that they have a common understanding on the function of bitcoin mining should play in the grid. There is always the problem of shortage possibility for renewable generation plants to provide minimum level of required demand. In this context, including bitcoin miners to energy buyer portfolio will help to improve the fundamentals of renewable energy generation, which has been fraught with volatility. Due to the second quarter Global Review 2021 report of Bitcoin Mining Council, 56% of the energy used for Bitcoin mining is provided by renewable resources. Moreover, aside from the environmental risks associated with these new types of currencies, there is also a view that the technology empowering them could also be a key enabler of the energy transition. Coherently we can sum up that there is a highly mutual relationship between cryptocurrencies, technology companies and renewable energy companies.

Many models such as average variance optimization, Markowitz's traditional portfolio analysis approach, omega measure-based optimization model have been tried in crypto asset-based strategy models. In this context, we will examine and compare the performances and total returns of crypto assets, technology stocks and gold according to Markowitz's traditional portfolio analysis approach. In this study, we aim to find the best portfolio distribution by comparing the risk, return and performance of crypto assets with renewables and technology shares.

2. LITERATURE REVIEW

The function of diversification is generally misunderstood or misinterpreted by practitioners. First, diversification is not a guarantee for a higher return compared to benchmark market returns. The main conclusion of the groundbreaking Modern Portfolio Theory (MPT) proposed by Markowitz (1952), the main purpose is not just to hold diversifies assets as portfolios, but also consider the relationship between the single assets of the portfolio. Treynor (1961), Sharpe (1964), and Lintner (1965) expanded the function of portfolios to determine the proper single asset premium. However, modern portfolio theory continues to play a critical role in the field of portfolio management, where the covariance and average of asset returns are still two major items of efficient portfolios. Given the covariance matrix and average returns, based on the preferred level of risk or targeted return values, optimal risky portfolio weights can be computed.

In this context, risk management is an important part of the portfolio selection process which also based on the profile and risk preference of the investor. The portfolio manager must be sure that the portfolio is consistent with the expectation of the investor. Since the portfolio manager will take a diversified portfolio perspective the more important part is how all the investments perform as a portfolio rather than the risk of any single asset. Moreover, cryptocurrencies' adoption increases globally along with other stable coins and new block chain based applications while governments such as China and India have forbidden bitcoin trading. Hence the largest near term risk is the uncertainty based on regulator actions. As a result, cryptos disturb the portfolio selection perspective with their very high risk-return characteristics compared to traditional financial assets.

Existing literature recently focus on price dynamics of Bitcoin as well as its relationship (cointegration, spillovers) with other financial assets. Co-movements among various asset classes can be materialized by indices which captures the integration between commodity and equity markets, (Silvennoinen and Thorp, 2013). Gatfaoui (2015) also highlights such relationships to be time varying which becomes a critical component in the portfolio construction stage of portfolio management.

The Chicago Board Options Exchange (CBOE) and Chicago Mercantile Exchange (CME) introduced futures contracts in December 2017, which is expected to rebuild the Bitcoin trading environment. According to Yermack (2015) the main reason for Bitcoin not being a useful unit of account is the high volatility of prices. Accordingly, Dyhrberg (2016) claims that due to its similar

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Table 1: Crypto assets in the portfolios of technology companies

Company Name	Symbol	Market Cap	% BTC	Basis Price USD	Today's Value	Bitcoin	%
Pub. Trad.							
MacroStrategy LLC (US)	NADQ: MSTR	$6,000,248,332	85	$3,159,863,658	$5,104,908,516	114,041 BTC	0.543
Tesla, Inc. (US)	NADQ: TSLA	$757,554,688,042	0.3	$1,500,000,000	$1,933,796,160	43,200 BTC	0.206
Square inc. (US)	NADQ: SQ	$113,959,131,394	0.3	$220,000,000	$359,319,023	8,027 BTC	0.038
Marathon Digital Holdings (US)	NADQ: MARA	$3,560,923,258	7	$161,539,500	$242,832,872	5,425 BTC	0.026
Coinbase Global, Inc. (US)	NADQ: COIN	$52,377,865,992	0	$242,832,872	$242,832,872	4,487 BTC	0.021
Hut 8 Mining Corp (CA)	NADQ: HUT	$1,779,395,000	11	$39,303,111	$199,198,910	4,450 BTC	0.021
Galaxy Digital Holdings (CA)	TSE: GLXY	$8,392,975,390	2	$179,055,200	$179,055,200	4,000 BTC	0.019
Bitcoin Group SE (DE)	ADE.DE	/	/	$179,055,200	$179,055,200	4,000 BTC	0.019
Bitfarms Limited (CA)	NASDAQ: BITF	$1,215,589,000	7.7	$16,817,350	$94,093,508	2,102 BTC	0.01
NEXON Co. Ltd (US)	TYO: 3659	/	/	$100,000,000	$76,859,445	1,717 BTC	0.008
Riot Blockchain, Inc. (US)	NADQ: RIOT	$2,841,518,953	2.5	$9,930,000	$70,055,347	1,565 BTC	0.007
Argo Blockchain PLC (US)	OTCPK: ARBKF	$514,519,024	11	$56,760,498	$56,760,498	1,268 BTC	0.006
Seetee AS (treasuries.NO)	AKER: NO	/	/	$58,599,450	$52,373,646	1,170 BTC	0.006
Meitu (treasuries.HK)	SEHK: 1357	/	/	$49,500,000	$42,118,233	941 BTC	0.004
Hive Blockchain (CA)	CVE: HIVE	$1,455,673,000	3	$39,168,325	$39,168,325	875 BTC	0.004
Coin Citadel Inc (US)	OTCMKTS: CCTL	/	/	$184,39	$22,963,829	513 BTC	0.002
Bit Digital, Inc. (US)	NADQ: BTBT	$566,954,758	3.9	$21,867,116	$21,867,116	489 BTC	0.002
Cypherpunk Holdings Inc. (CA)	CSE: HODL	$21,124,057	76	$5,637,663	$16,106,015	360 BTC	0.002
BIGG Digital Assets Inc. (CA)	CNSX: BIGG	$218,806,112	6	$2,690,387	$13,429,140	300 BTC	0.001

Source: www.buybitcoinworldwide.com

Table 2: Crypto assets in the portfolios of technology companies

Company Name	Symbol	Market Cap	% BTC	Basis Price USD	Today's value	Bitcoin	%
ETF Like							
Digihost Technology Inc. (CA)	TSXV: DGHI.V	/	/	$6,890,000	$8,223,110	184 BTC	0.001%
Fortress Blockchain (CA)	TSXV: FORT	/	/	$7,305,452	$7,305,452	163 BTC	0.001%
CleanSpark Inc (US)	NASDA: CLSK	$457,347,704	1	$6,401,223	$6,401,223	143 BTC	0.001
Banxa Holdings Inc (CA)	OTCMKTS: BNXAF	$150,037,800	4	$6,087,877	$6,087,877	136 BTC	0.001
Brooker Group's (treasuries.TH)	SET: BROOK	/	/	$6,599,916	$5,475,320	122 BTC	0.001
Neptune Digital Assets (CA)	TSX-V: NDA	/	/	$4,476,380	$4,476,380	100 BTC	0
Mode Global Holdings (UK)	LON: MODE	/	/	$975,09	$3,800,277	85 BTC	0
BTCS Inc. (US)	OTCQB: BTCS	/	/	$3,515,480	$3,515,480	79 BTC	0
FRMO Corp. (US)	OTCMKTS: FRMO	$453,331,600	0.6	$2,812,957	$2,812,957	63 BTC	0
QwD FinTech Corp (CA)	TSXV: LQWD	/	/	$2,280,000	$2,685,828	60 BTC	0
MOGO Financing (CA)	NADQ: MOGO	$417,755,900	0.5	$2,238,190	$2,238,190	50 BTC	0
Phunware, Inc. (US)	NADQ: PHUN	$77,865,005	1.5	$1,499,831	$1,154,906	26 BTC	0
Globant S.A. (US)	NYSE: GLOB	$13,457,794,832	0	$500,00	$671,46	15 BTC	0
BlackRock (US)	NYSE: BLK	$139,505,713,452	0	$360,00	$275,30	6.15 BTC	0
Net Holding Anonim Sirketi (treasuries.TR)	IST: NETHL	/	/	$79,80	$125,34	2.8 BTC	0
MTGOX K.K. (JP)	private	/	/	$68,576,024	$6,342,403,767	141,686 BTC	0.675
Block.one (US)	private	/	/	$7,341,263,200	$7,341,263,200	164,000 BTC	0.781
The Tezos Foundation (CH)	private	/	/	$1,110,500,350	$1,110,500,350	24,808 BTC	0.118
Stone Ridge Holdings Group (US)	private	/	80	$115,000,000	$555,230,110	10,889 BTC	0.052
Bulgaria (treasuries.BG)	gov	/	/	$3,300,000,000	$9,557,921,812	213,519 BTC	1017
Ukraine (various) (treasuries.UA)	gov	/	/	$2,408,346,974	$2,408,346,974	46,351 BTC	0.221

(Contd...)

Table 2: (*Continued*)

Company Name	Symbol	Market Cap	% BTC	Basis Price USD	Today's value	Bitcoin	%
El Salvador (treasuries.ES)	gov	/	/	$25,872,220	$25,872,220	550 BTC	0.003
Georgia (treasuries. GE)	gov	/	/	$2,954,411	$2,954,411	66 BTC	0
Grayscale Bitcoin Trust (US)	OTCQX: GBTC	$25,479,220,000	115	$29,315,149,345	$29,315,149,345	654,885 BTC	3.12
CoinShares/XBT Provider (EU)	COINXBT: SS	/	/	$770,170,932	$3,121,379,774	69,730 BTC	0.332
3iQ The Bitcoin Fund (CA)	TSX: QBTCU	$653,408,400	/	$1,050,113,984	$1,050,113,984	23,459 BTC	0.112
Purpose Bitcoin ETF (CA)	TSX: BTCC (U/B)	/	/	$905,171,432	$905,171,432	20,221 BTC	0.096
3iQ CoinShares Bitcoin ETF (CA)	TSX: BTCQ	/	/	$816,581,240	$816,581,240	18,242 BTC	0.087
ETC Group Bitcoin ETP (DE)	BTCE: GR	/	/	$779,830,160	$779,830,160	17,421 BTC	0.083
Bitwise 10 Crypto Index Fund (US)	OTCQX: BITW	/	/	$611,009,429	$611,009,429	13,650 BTC	0.065
21Shares AG (CH)	multiple	/	/	$347,786,747	$347,786,747	7,769 BTC	0.037
Grayscale Digital Large Cap Fund (US)	OTCMKTS: GDLC	/	/	$329,864,442	$329,864,442	7,369 BTC	0.035
Ninepoint Bitcoin Trust (CA)	TSX: BITC.U	/	/	$212,646,383	$308,165,026	6,884 BTC	0.033
Hashdex Nasdaq Crypto Index Fundo de Indice (treasuries. BR)	BVMF: HASH11	/	/	$269,237,710	$269,237,710	6,015 BTC	0.029
WisdomTree Bitcoin (CH)	BTCW: SW	/	/	$255,146,432	$255,146,432	5,700 BTC	0.027
CI Galaxy Bitcoin Fund (CA)	TSX: BTCG.U	/	/	$177,659,497	$177,659,497	3,969 BTC	0.019
VanEck Vectors Bitcoin ETN (US)	XETRA: VBTC	/	/	$177,290,769	$177,290,769	3,961 BTC	0.019
Leonteq Bitcoin Tracker USD (CH)	UBTCTQ	/	/	$106,790,272	$106,790,272	2,174 BTC	0.01
Evolve Bitcoin ETF (CA)	TSX: EBIT	/	/	$95,775,000	$95,775,000	2,140 BTC	0.01
Osprey Bitcoin Trust (US)	OBTC	/	/	$81,564,835	$81,564,835	1,639 BTC	0.008

Source: www.buybitcoinworldwide.com

hedging capabilities with gold, Bitcoin can be utilized as a hedging tool against the UK stock market and US dollar.

Applying a quantile regression method, Bouri et al. (2017) investigate the relationship between gold and global uncertainty concluding that Bitcoin can also be an alternative hedging tool against global uncertainty at short term investment period. Brauneis and Mestel (2019) analyze risk-return benefits of portfolios which includes cryptocurrency by utilizing a Markowitz mean-variance framework. In the study they conclude that combining cryptocurrencies provides lower-risk for investment portfolios containing crypto assets. While other studies mostly focus on the diversification effect of adding one single cryptocurrency (usually Bitcoin) to a portfolio containing conventional asset classes, Brauneis and Mestel find solid potential for risk reduction when several cryptocurrencies are mixed. Platanakis and Urquhart (2020) study the potential out-of-sample benefits of adding Bitcoin to a stock-bond portfolio. Based on three different levels of risk aversion, their study depends on various asset allocation strategies in which they conclude that the out-of-sample benefits

of Bitcoin are consistent across different levels of risk aversion and portfolio construction approaches. Although it may still be an effective diversifier, limited evidence of the hedging and relatively safe features of Bitcoin is presente by Bouri et al. (2017) who apply a DCC model in their study. Klein et al. (2018) compare commodities, cryptocurrencies and equity indices in terms of volatility behavior. According to the results of their study, Bitcoin cannot be named as the new Gold, which is also consistent with the argument of our own study. With a similar approach, Henriques and Sadorsky (2018) employ Modern Portfolio Theory and replace gold with Bitcoin in an investment portfolio to analyze possible effects of this replacement. According to their results although substituting gold for Bitcoin in a portfolio is possible it provides a high-risk adjusted return. Sarkodie et al. (2022) focus on the impact of COVID-19 health issues on Bitcoin, Ethereum, Bitcoin, Cash, and Litecoin prices concluding that investors need to diversify their investment to avoid selling out whole portfolio during recession of markets because of a possible rebound-effect related with cryptocurrencies. Lopez-Cabarcos et al. (2019) show that Bitcoin volatility is highly unstable in speculative periods while in stable

periods S&P 500 and fear index (VIX) are main influencers of its volatility. In this study they utilize GARCH and EGARCH models.

On the other hand another major issue about cryptocurrencies is production cost and energy consumption for mining process as well as its environmental impact. Based on the amount of electricity required in mining, production costs are relatively high for both crypto asset and commodity markets which involve mining, however, renewable energy resources gradually reduce the cost by providing cheaper energy. Both commodity markets and crypto currency can be used as a store of value considering demand elements, however, cryptocurrencies are extremely sensitive to exchange rate risk as well as commodity price risk. Cryptos have neither intrinsic values nor they can be used as units of account while commodities have both of these two properties. Another important issue is the price bubbles. Due to their highly speculative features, cryptocurrency assets are subject to abnormal returns for a small proportion of the investors which avoids cryptocurrency markets to create any real value for real economies and society.

As a result its high-energy consumption is seriously argued to be unnecessary, unsustainable and even wasteful. However, Bitcoin supporters highlight that in 2020, 76% of digital asset miners used renewable energy based resources during their production process. Moreover they also counter that the energy consumption is worth for improved financial inclusion, democratization of finance, and their role as store of value since fiat currencies are devalued especially in recent high inflation economic environment.

In Canada, the government had to stop further requests of power from crypto miners who consume too much energy with their mining processes (Meyer, 2018). Moreover, Gurrib (2019) finds energy spot markets tend to move together, with energy based crypto currencies. Chuen et al. (2018) utilized crypto index CRIX and reported low correlations with other commodities like gold, however, found a negative relationship between crude oil and energy cryptos. Investors do not contribute to mining process directly but still Bitcoin allocations increase the carbon footprints of their existing portfolios. Even if investors do not construct a portfolio consisting of only cryptocurrencies, they still have negative impact on the environment indirectly.

3. DATA AND ANALYSIS

In our models we use daily market data, for a period between January 01, 2016 and September 30, 2021. We collected the data from www.investing.com which is freely available for researches who want to replicate our results and approach. Our data set includes two major cryptocurrencies, Bitcoin and Etherium. We relate risk and return of different mean-variance portfolio strategies to Bitcoin (BTC), Etherium (ETH), S&P Global Clean Energy Index (SPGCE) and MSCI World Information Technology Index (MSCIWIT).

We start our analysis by illustrating the fundamental characteristics of our data in Table 3 which shows the descriptive statistics for the returns of BTC, ETH, SPGCE and MSCIWIT. For all the returns series the mean values are close to zero while cryptocurrencies

Table 3: Descriptive statistics

	RBTC	RETH	RMSCIWIT	RSPGCE
Mean	0.0037	0.0045	0.0013	0.0014
Median	0.0022	0.0025	0.0022	0.0023
Maximum	0.1938	0.3492	0.1015	0.1069
Minimum	–0.4809	–0.5925	–0.1016	–0.1207
Std. Dev.	0.0481	0.0631	0.0146	0.0190
Skewness	–1.3819	–1.1782	–0.5593	–0.9590
Kurtosis	18.8068	17.1136	10.6303	11.4741
Jarque-Bera	7499.5	5963.3	1732.2	2198.6
Probability	0.0000	0.0000	0.0000	0.0000
ADF Test Level	–28.772	–29.320	–27.379	-8.583
	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Between parenthesis: P values. The number of observations is 699. ADF tests refer to Augmented Dickey Fuller test for the presence of unit root for long differences (returns)

are still more clustered compared to MSCIWIT and SPGCE. Each return has typical characteristics of leptokurtosis and fat-tail since in common the skewness of each return is not equal to zero and neither is the kurtosis. Nevertheless, typical characteristic of financial time series is that they are mostly leptokurtosis and fat-tailed. Considering the J-B statistics of each returns which are significantly different from zero, we can clearly conclude the returns do not fit to the normal distribution. The stationarity issue is an important item while dealing with time series models. Although, using return series generally saves us from this trouble, we still employ Augmented Dickey-Fuller (ADF) unit root test and examine the stationarity of the variables. As a result, the null hypothesis of the unit root is strongly rejected for all return series due to ADF test level values in Table 3.

In Figure 1 we can see the return clusters BTC, ETH, MSCIWIT and SPGCE. Time series exhibit hike of crypto assets after December 2020 which drives us to examine high volatile nature of these assets. This high volatility environment require portfolio managers re-optimizate their asset allocation strategies to catch the ultimate optimal portfolio reflecting the fluctuating feature of crypto markets. The high volatility which is clear on the 300th observation of all series is the 2020 stock market crash. It was a major and sudden global stock market crash that began on 20 February 2020 and ended on 7 April. During the crash, there were multiple severe daily drops in the global stock market.

However in Figure 2, combined daily return graphs of BTC, ETH, MSCIWIT and SPGCE show that the volatility of BTC (blue line) and ETH (red line) is too high compared to MSCIWIT and SPGCE which makes us to consider how to treat these assets while constructing an investment portfolio. These high volatile assets make it harder for portfolio managers to combine cryptocurrencies with traditional financial assets since mean reverting approaches like Markowitz has limits to solve the optimization problem without any constraints. In the empirical results part we analyze this issue with six different cases by constructing portfolios with various constraints.

4. METHODOLOGY

In our study, firstly, we will apply VAR-VECH-TARCH models to analyze the spillover relationship between BTC, ETH,

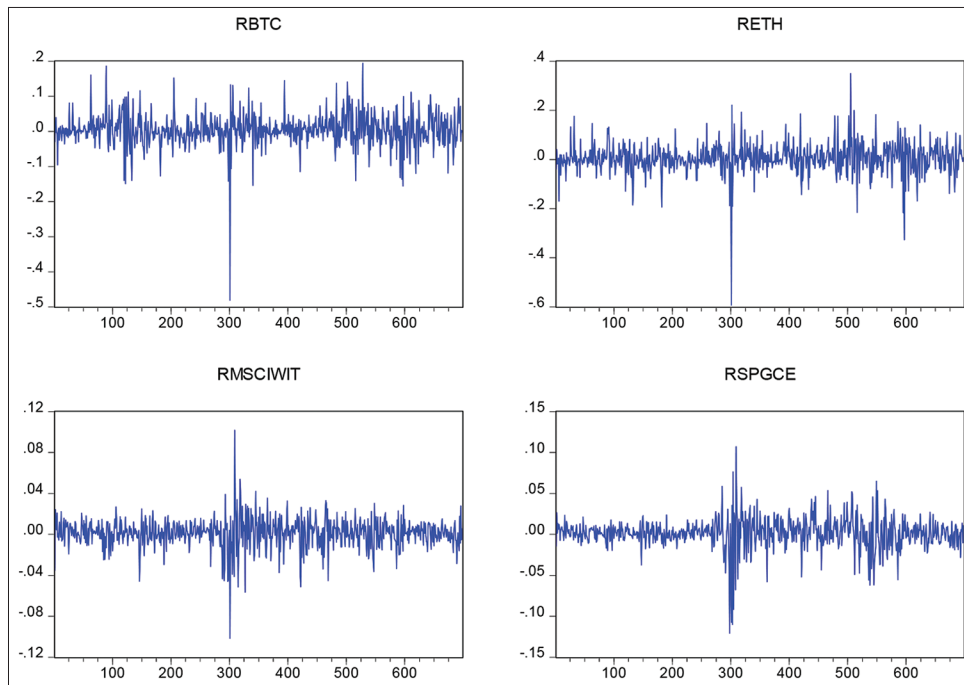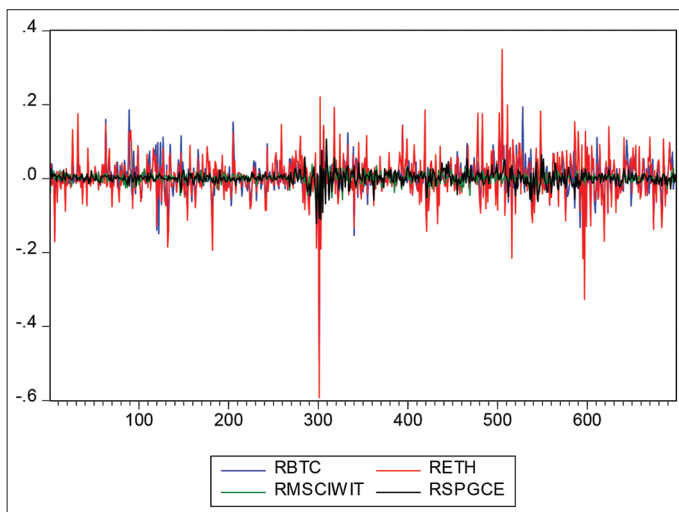**Figure 1:** Daily return graphs of Bitcoin, Etherium, MSCIWIT and SPGCE

Figure 2: Combined daily return graphs of Bitcoin, Etherium, MSCIWIT and SPGCE

MSCIWIT and SPGCE. Secondly, we will apply the Markowitz portfolio selection approach to understand and present risk-return benefits for portfolios which includes cryptocurrency. Whether diversification is still an efficient tool for portfolios including assets with very high volatility features is the major research question of this paper.

4.1. VAR-VECH-TARCH Models

There is a significant accumulated literature work focusing on the asymmetric responses to positive and negative information such that bad news has a much greater and persistent impact on market reactions than does good news. As a typical reference, the effects of good and bad news to have different impact on volatility (Glosten et al., 1993). Briefly if we define $\varepsilon_{t-1} = 0$ as a threshold where shocks greater than the threshold has different effects than

shocks below the threshold. This method is mainly a modified version of VAR-GARCH which is proposed by Ling and McAleer (2003). This structure enables us to examine the conditional returns and conditional volatility with meaningfully estimated parameters.

Here we consider the threshold-GARCH (TARCH) process:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \lambda_1 d_{t-1} \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{1}$$

where d_{t-1} is a dummy variable that is equal to one if $\varepsilon_{t-1} < 0$ and is equal to zero if $\varepsilon_{t-1} \geq 0$. The intuition behind the TARCH model is that positive values of ε_{t-1} are associated with a zero value of d_{t-1}. Hence if $\varepsilon_{t-1} \geq 0$, the effect of an ε_{t-1} shocks on h_t is ε_{t-1}^2. When $\varepsilon_{t-1} < 0$, $d_{t-1} = 1$, and the effect of an ε_{t-1} shock on h_t is $(\alpha_1 + \lambda_1) \varepsilon_{t-1}^2$. If $\lambda_1 > 0$, negative shocks will have larger effects on volatility than positive shocks. Basically, this method is composed of two parts, VAR model and asymmetric VECH-TARCH model. This combination allows the researchers to explore the joint evolution of conditional returns and volatility spillovers between different financial markets in a compact framework.

In this context, firstly, the univariate autoregressive (AR) is extended to the vector autoregressive (VAR) by incorporating the related variables into endogenous variables. Hence, VAR models examine the contagion and spillover effect between major financial markets.

The basic mathematical expression of the VAR model is as follows:

$$R_t = C + A_1 R_{t-1} + A_2 R_{t-2} + \cdots + A_k R_{t-k} + \varepsilon_t \tag{2}$$

$$\varepsilon_t I_{t-1} \sim N(0, H_t)$$

where R_t refers to the value of endogenous variables vector at time t, C is the constant vector, matrix A is the estimated coefficients

and k is the lag operator. Residual vector ε_t is assumed to be normally distributed with a zero mean and constant variance where the market information available at time $t–1$ denoted as $d_{t-1.}$ The lag order of (k) VAR structure is decided via AIC criterion, FPE criterion, and LR.

In this approach, we incorporate a three-dimensional model to examine the news spillover between different markets. Suppose that our model structure is as follows:

$$\varepsilon_{i,t}=v_{i,t}.h_{i,t}, \; v_{i,t}\sim N(0,1) \tag{3}$$

$$h_{i,t}=c_i+a_i \, \varepsilon_{t-1}{}^2+\beta_i \, h_{i,t-1} \tag{4}$$

$$H_t=C^T \, C+A^T \, \varepsilon_{t-1} \, \varepsilon_{t-1}{}^T \, A+B^T \, H_{t-1} \, B \tag{5}$$

Equation (3) specifies the relation between the residual term $\varepsilon_{i,t}$ and the conditional variance $h_{i,t}.v_{i,t}$ which is normally distributed with a zero mean and constant variance. α, β are the coefficients. $H_{i,t}$ represents the conditional variance-covariance matrix, C represents the lower triangular matrix, A and B are square arrays. If $C^T C$ is positive, then it is almost positive.

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} & h_{13,t} \\ h_{12,t} & h_{22,t} & h_{23,t} \\ h_{31,t} & h_{32,t} & h_{33,t} \end{bmatrix}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

where $h_{11,t}$, $h_{22,t}$, $h_{33,t}$ in the matrix H_t represent the conditional variances. Matrix A is the ARCH coefficients of the model, a_{11}, a_{22}, a_{33} represent the ARCH effect while Matrix B is the GARCH coefficients of the model, b_{11}, b_{22}, b_{33} are the GARCH effect.

In consideration of the asymmetric effect diagonal VECH is:

$$H_t = A_0 + \sum_{i=1}^{p} A_i \otimes H_{t-i} + \sum_{i=1}^{q} B_i \otimes \varepsilon_{t-1}\varepsilon_{t-1}^T \tag{6}$$

where the conditional variance covariance equation of a bivariate (VECH) TARCH model has the following form:

$$VECH(H_t)=C+AVCEH(\varepsilon_{t-1} \varepsilon_{t-1}{}')+BVECH(H_{t-1}) H_{t-1}{}')+DVECH(\varepsilon_{t-1}\varepsilon_{t-1}{}')(d_{t-1}) \tag{7}$$

where the last term on the RHS of equation (7) depicts the asymmetries. In this context the diagonal bivariate VECH model is as follows:

$$h_{11,t}=C_{01}+a_{11} \, \varepsilon_{1,t-1}{}^2+b_{11} \, h_{11,t-1} \tag{8}$$

$$h_{12,t}=C_{02}+a_{33} \, \varepsilon_{1,t-1}\varepsilon_{2,t-1}+b_{22} \, h_{12,t-1} \tag{9}$$

$$h_{22,t})=C_{03}+a_{33} \, \varepsilon_{2,t-1}{}^2+b_{33} \, h_{22,t-1} \tag{10}$$

The coefficient a_{11} refers to the ARCH process in the residuals from asset i which depicts the fluctuations of the assets reflecting the impact of external shocks on fluctuations. The ARCH effects measure short-term persistence while the GARCH effect measure long-term persistence. The coefficient represents the ARCH process in the second asset residuals. The parameters between asset i and asset j. The calculation of the time-varying beta coefficient is done as

$$\beta_{it}^{BG} =\hat{h}_{12,t} / \hat{h}_{22,t} \tag{11}$$

where the symbol ^ indicates the estimated values of conditional variance.

4.2. Markowitz Mean-variance Framework

In addition to spillover analysis via VAR-VECH-TARCH models, we also rely on portfolio selection framework as proposed by Markowitz (1952) to address and quantify portfolio effects in the crypto-asset universe. For both econometric modeling and portfolio allocation applications we calculate daily log-returns derived from closing prices (P) such that:

$$R_{it}=ln(P_t)–ln(P_{t-1}).$$

Estimating an investor risk averse, Markowitz portfolio theory lets us to analyze the efficiency of fund allocation to selected assets based on means and the variance of the returns.

Let us consider a portfolio with n different assets where asset number i will give the return R_i and mean, and variance will be represented with μ_i and σ_i^2. The covariance between R_i and R_j. Moreover, x_i will refer to the portion of the value of the amount invested in asset i. If R is the return of the whole portfolio:

$$\mu = E[R]= \sum_{i=1}^{n} \mu_i x_i \tag{1}$$

$$\sigma^2 = Var[R]= \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j}x_i x_j \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{3}$$

For different weights of $x_1, ..., x_n$ the investor will be provided different combinations of μ and σ^2. However, in Markowitz framework one of the assumptions is that short sales is not allowed which requires us one more condition such as:

$$x_i \geq 0, \; i = 1, 2,…, n \tag{4}$$

In this context, equations (4) confirms that only long positions are allowed in Markowitz portfolio diversification approach.

5. EMPIRICAL RESULTS

In Table 4, we can see the results of the VAR-VECH-TARCH models. The own conditional ARCH effects (a_{ii}) are significant at the %1 level for all assets. Moreover, in the short term, volatility

Table 4: Estimation results of asymmetric VAR-VECH-TARCH (1,1) models: Transformed variance coefficients

Model 1	Coefficient	z-Statistic	P-value
M (1,1)	0.0003***	0.0000	0.0000
M (1,2)	0.0002***	0.0000	0.0000
M (1,3)	0.0000	0.0000	0.2975
M (1,4)	0.0000	0.0000	0.1257
M (2,2)	0.0003***	0.0000	0.0000
M (2,3)	0.0000	0.0000	0.4036
M (2,4)	0.0000*	0.0000	0.0904
M (3,3)	0.0000***	0.0000	0.0005
M (3,4)	0.0000***	0.0000	0.0033
M (4,4)	0.0000***	0.0000	0.0033
A1 (1,1)	0.1622***	0.0202	0.0000
A1 (1,2)	0.0944***	0.0143	0.0000
A1 (1,3)	0.0211	0.0373	0.5708
A1 (1,4)	−0.0535*	0.0348	0.1240
A1 (2,2)	0.0734***	0.0150	0.0000
A1 (2,3)	0.0244	0.0382	0.5229
A1 (2,4)	−0.0160***	0.0153	0.2934
A1 (3,3)	0.0817***	0.0330	0.0133
A1 (3,4)	0.0710***	0.0278	0,0107
A1 (4,4)	0.1158***	0.0230	0.0000
D1 (1,1)	−0.0199***	0.0008	0.0000
D1 (1,2)	0.0049	0.0150	0.7444
D1 (1,3)	−0.0005	0.0379	0.9891
D1 (1,4)	0.0005	0.0518	0.9925
D1 (2,2)	−0.0030	0.0209	0.8852
D1 (2,3)	0.0004	0.0493	0.9941
D1 (2,4)	−0.0001	0.0181	0.9963
D1 (3,3)	0.0004	0.0352	0.9900
D1 (3,4)	0.0001	0.0318	0.9974
D1 (4,4)	−0.0001	0.0268	0.9970
B1 (1,1)	0.7603***	0.0274	0.0000
B1 (1,2)	0.8200***	0.0194	0.0000
B1 (1,3)	0.8257***	0.1487	0.0000
B1 (1,4)	0.5728***	0.2367	0.0155
B1 (2,2)	0.8662***	0.0159	0.0000
B1 (2,3)	0.7383***	0.2850	0.0096
B1 (2,4)	0.9290***	0.0296	0.0000
B1 (3,3)	0.8699***	0.0281	0.0000
B1 (3,4)	0.8876***	0.0253	0.0000
B1 (4,4)	0.8779***	0.0172	0.0000

***, **,** represent 1%, 5% and 10% significance respectively. In Model 1, Bitcoin, Etherium, MSCI World Information Technology Index and S&P Global Clean Energy Index are represented by 1,2 and 3

spillover exists between BTC and ETH since a_{12} is significant even at 1% level while volatility spillover also exists between ETH-MSCIWIT and MSCIWIT-SPGCE since a_{24} and a_{34} are significant at the 1% level. However, asymmetry does not exist since $d_{12}, d_{13}, d_{14}, d_{23}, d_{24}$ and d_{34} are all statistically insignificant. In the long-term volatility spillover exists among all assets since $b_{12}, b_{13}, b_{14}, b_{23}, b_{24}$ and b_{34} are all statistically significant in all models.

In Figures 3 and 4 we can see the covariance and correlation coefficients of all assets based on the model represented in Table 4. Conditional variance is all time high on March 2020 market crash. Conditional correlation range between BTC and ETH is much higher compared to other pairs in the second quarter of 2021.

A rational and risk averse investor targets a high profit and a small risk. In this context his/her objective function is either to

maximize μ or minimize σ^2. Therefore, the portfolio manager should optimize the assets' allocation and select a portfolio which gives a (σ^2, μ) combination in the efficient investment set. Hence in our study we defined three major objective function of the portfolio manager such as;

- To minimize portfolio annual variance (standard deviation)
- To maximize portfolio annual return

Here we should note that just targeting a return maximization or variance minimization may not be efficient for portfolios including assets with very high volatility such as cryptocurrencies. Hence, including a more balanced approach including relevant ratios like Sharpe ratio may add value to the analysis. As a result we add third objective as;

- To maximize Sharpe Ratio

Since without any weight constraints, the high returns of cryptocurrencies experienced in recent years may crowd-out SPGCE and MSCIWIT from portfolio. So we included various constraints to return maximization objectives to provide a more diversified portfolio samples. Here our main aim is to see the change of asset allocation and weights of the individual assets produce by Markowitz optimization. Otherwise since cryptocurrency data includes a tremendous return jump in historical data, this ex-ante approach may misguide us to understand that interaction among all selected assets. With the same approach, we also included constraints for minimization objectives as well to avoid crowding out of the cryptocurrencies because of their quite high volatile features. Based on the historical performances even ETH crowds out BTC in some cases without any additional allocation constraint.

As the result of our optimization problem sets and Markowitz framework applications, Table 5 represents the results for a set of portfolios with and without any extra constraints for an objective function to maximize Sharpe ratios[3] of the portfolios.

If we do not add any constraint the return of cryptocurrencies is ×2.68, ×2.87 times more for BTC and ×3.26, ×3.50 times more for ETH respectively compared to SPGCE and MSCIWIT which drives fund managers to include only ETH to maximize returns (Case 2) and to minimize the risk the fund managers choose to include only MSCIWIT (Case 3) without any constraints. Consequently if the objection function is to maximize Sharpe ratio[4] without any constraints, fund managers allocate investment budget such as %19 BTC, 10% ETH, 18% SPGCE and 54% MSCIWIT (Case 1).

However, the portion of BTC, ETH and MSCIWIT and SPGCE vary due to different constraints. Yet adding another constraint such as telling the fund manager that the costumer wants to include at

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3   The Sharpe ratio was developed by Nobel laureate William F. Sharpe and is used to help investors understand the return of an investment compared to its risk. The ratio is the average return earned more than the risk-free rate per unit of volatility or total risk. Volatility is a measure of the price fluctuations of an asset or portfolio.

4   Risk-free rate taken for Sharpe ratio calculation is %19 coherent with the benchmark policy rate of Turkish Central Bank

Table 5: Portfolio *sets and asset allocations*

Case 1			Case 2			Case 3		
Objective: Maximize Sharpe ratio, Constraint: No constraint			**Objective: maximize return, Constraint: No constraint**			**Objective: minimize volatility, Constraint: No constraint**		
Portfolio Weights			Portfolio Weights			Portfolio Weights		
RBTC	19%		RBTC	0%		RBTC	0%	
RETH	10%		RETH	100%		RETH	0%	
RSPGCE	18%		RSPGCE	0%		RSPGCE	0%	
RMSCIWIT	54%		RMSCIWIT	0%		RMSCIWIT	100%	
	1.00			1.00			1.00	
	Daily	**Annualized**		**Daily**	**Annualized**		**Daily**	**Annualized**
Return	0.21%	76%	Return	0.45%	165%	Return	0.13%	47%
Variance	0.04%		Variance	0.40%		Variance	0.02%	
Std. Dev	1.98%	38%	Std. Dev	6.31%	121%	Std. Dev	1.46%	28%
Risk-free rate	19%		Risk-free rate	19%		Risk-free rate	19%	
Sharpe Ratio	1.502869164		Sharpe Ratio	1.207391313		Sharpe Ratio	1.006575129	
Case 4			Case 5			Case 6		
Objective: Maximize Sharpe ratio, Constraint: Minimum %10 investment to each asset			**Objective: maximize return, Constraint: Minimum %10 investment to each asset**			**Objective: minimize volatility, Constraint: No constraint**		
Portfolio Weights			Portfolio Weights			Portfolio Weights		
RBTC	19%		RBTC	10%		RBTC	10%	
RETH	10%		RETH	70%		RETH	10%	
RSPGCE	18%		RSPGCE	10%		RSPGCE	12%	
RMSCIWIT	54%		RMSCIWIT	10%		RMSCIWIT	68%	
	1.00			1.00			1.00	
	Daily	**Annualized**		**Daily**	**Annualized**		**Daily**	**Annualized**
Return	0.21%	76%	Return	0.38%	139%	Return	0.19%	68%
Variance	0.04%		Variance	0.24%		Variance	0.03%	
Std. Dev	1.98%	38%	Std. Dev	4.90%	94%	Std. Dev	1.74%	33%
Risk-free rate	19%		Risk-free rate	19%		Risk-free rate	19%	
Sharpe Ratio	1.502822118		Sharpe Ratio	1.277068085		Sharpe Ratio	1.475394422	

Figure 3: Conditional variance graphs
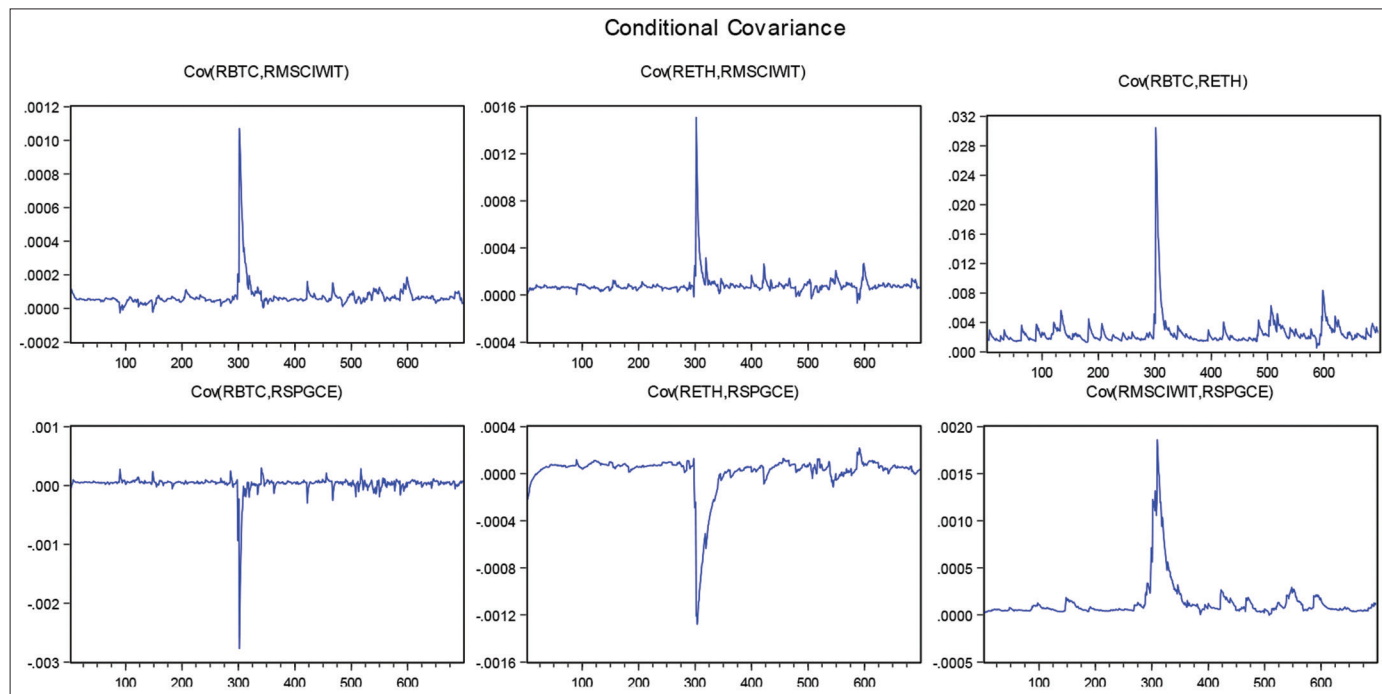
least 10% investment to all assets the results change in Cases 4, 5 and 6. The optimal weights are calculated as 19% BTC, 10% ETH, 18% SPGCE and 54% MSCIWIT to maximize returns (Case 4) while these weights change to 10% BTC, 70% ETH, 10% SPGCE and 10% MSCIWIT to maximize Sharp Ratio (Case 5). Finally if we change the objective function to minimize portfolio risk,

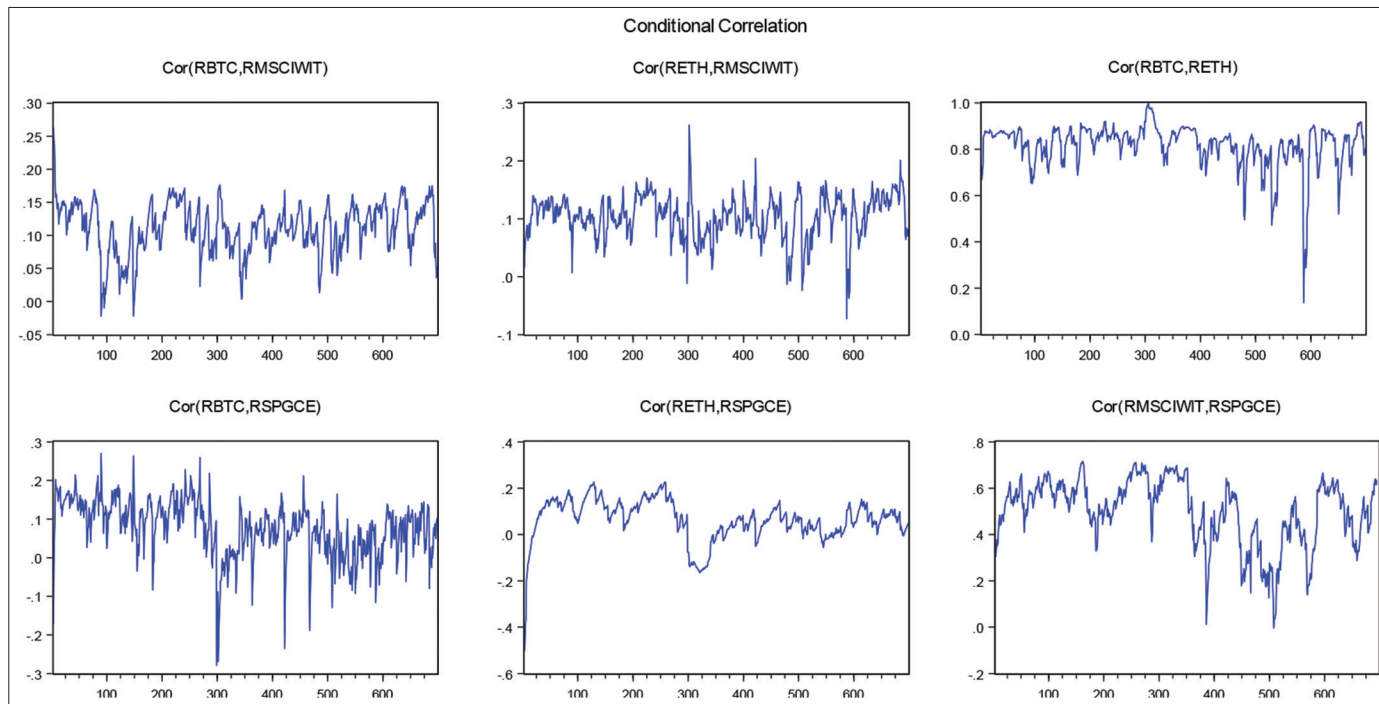**Figure 4:** Conditional correlation graphs
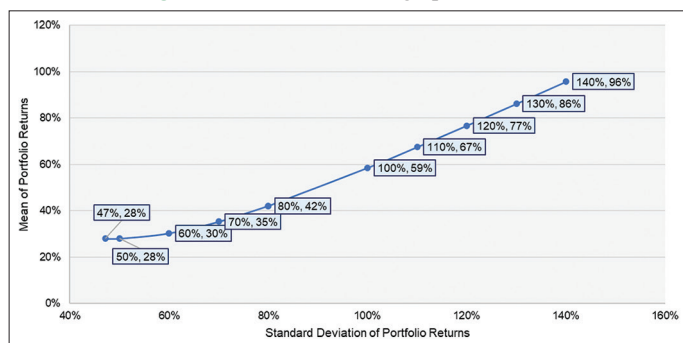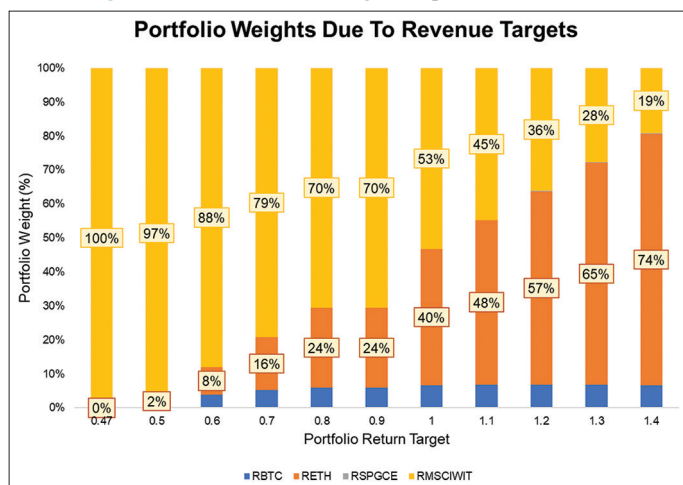

Figure 5: Efficient frontier graph for Case 2

Figure 6: Portfolio asset weights in porfolio for Case 2

With the perspective of Markowitz Portfolio Theory, the efficient frontier of various portfolios without any constraints is illustrated in Figure 5. An important point in Figure 5 is that the illustrated frontier is based on the targeted revenues with an objection function to minimize risk without any asset weight constraints as well as Since the portfolio manager or individual investors have a tendency to be risk averse.

Finally, when we increase the target revenue, the portion of the cryptocurrencies included to the portfolio also increases due to portfolio optimization based on the historical performance of the assets (Figure 6). As a result even if the investor is defined as a risk-averse profile, if your objective function is to increase your target revenue with a portfolio composed of BTC, ETH, SPGCE and MSCIWIT you have to increase cryptos' portion in the portfolio. Of course, these portions will be adjusted and most probably there will be a substitution between BTC and ETH based according to various cases in Table 5. Since this outcome is crystal clear we did not perform such an exercise to keep the refined feature of the paper.

6. CONCLUSION

Compared to traditional financial assets major crypto-assets' volatility is still high but they give signs of maturing. Investors can utilize traditional portfolio allocation approaches to adjust their asset allocation by weighting volatility inversely. Obviously with this approach assets with higher volatility will occupy a lower portion in the portfolio. Another aspect of cryptocurrencies is its mutual relationship with energy, especially renewables, companies. Many energy companies started to invest in cryptocurrencies suggesting that they have a shared understanding

the optimal weights are calculated as 10% BTC, 10% ETH, 12% SPGCE and 68% MSCIWIT (Case 6).

of the role bitcoin mining should play in the grid. Moreover, aside from the environmental risks associated with these new types of currencies, there is also a view that the technology empowering them could also be a key enabler of the energy transition so there is a highly mutual relationship between cryptocurrencies, technology companies and renewable energy companies.

Actually the variance of a portfolio is not an ultimate measure of the risk taken by the investor which is one of the most important weakness of Markowitz framework. This approach does not tell any investor clearly which portfolio he/she shall invest if he/she is targets a certain high-level risk. Although Markowitz model has limitations, we find this exercise useful to figure out different characteristics of cryptoassets and various exchange indices.

According to our results without any specific constraint for maximization or minimization problems Markowitz optimization approach totally crowds out some of the assets among cryptocurrency or clean energy and technology companies based on the objective function. Our next attempt will cover alternative portfolio optimization approaches for further research in this field.

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